HIGH ENERGY RINGS

initiated at Snowmass 96

Last Modified: 4/9/98 MJS
5/1/97 MJS/MH
Modified by Saeed - 1/2/2014
MathCad Prime V3.0

CONSTANTS and UNITS

$q := 1.60217733 \cdot 10^{-19} \cdot coul$

$msec := 10^{-3} \cdot sec$

$Hz := sec^{-1}$ $\quad \mu sec := 10^{-6} \cdot sec$ $\quad nsec := 10^{-9} \cdot sec$ $\quad psec := 10^{-12} \cdot sec$

$c := 2.99792458 \cdot 10^{8} \cdot \frac{m}{sec}$

$kHz := 10^{3} \cdot Hz$ $\quad MHz := 10^{6} \cdot Hz$ $\quad MJ := 10^{6} \cdot joule$

$eV := q \cdot volt$ $\quad keV := 10^{3} \cdot eV$ $\quad MeV := 10^{6} \cdot eV$ $\quad GeV := 10^{9} \cdot eV$ $\quad TeV := 10^{12} \cdot eV$

$hbar := 6.5821220 \cdot 10^{-22} \cdot MeV \cdot sec$ $\quad \mu m := 10^{-6} \cdot m$ $\quad Agstm := 10^{-10} \cdot m$ $\quad mr := 10^{-3}$

$\mu r := 10^{-6}$

$E_0 := 0.93827231 \cdot GeV$ $\quad Mohm := 10^{6} \cdot ohm$

$mb := 10^{-31} \cdot m^2$ $\quad pb := 10^{-9} \cdot mb$

$\mu_0 := \frac{4 \cdot \pi}{10^7} \cdot \frac{tesla \cdot m}{amp}$ $\quad kwatt := 10^{3} \cdot watt$

$GJ := 10^{9} \cdot joule$ $\quad ton\_TNT := 4184 \cdot MJ$

$\varepsilon_0 := \frac{1}{c^2 \cdot \mu_0}$ $\quad doz\_jelly\_doughnuts := 12 \cdot 500 \cdot 4200 \cdot joule$

ACCELERATOR INPUT PARAMETERS

$C := 1868 \cdot km$ — **Ring Circumference**

$E_{ring} := 250 \cdot TeV$ — **Collider Top Energy**

$E_{inj} := 50 \cdot TeV$ — **Injection Energy**

$\beta x_{star} := 50 \cdot cm$

$\beta y_{star} := 50 \cdot cm$ — **Beta Star (hor/ver) at each IP (collision optics)**

$B := 200000$ — **Number of bunches**

$N_0 := 6 \cdot 10^{10}$ — **Initial Bunch intensity**

$\varepsilon x_{n_0} := 2 \cdot \pi \cdot mm \cdot mr$ — **Initial transverse emittances (normalized, rms)**

$\varepsilon y_{n_0} := 2 \cdot \pi \cdot mm \cdot mr$

$S := 0.8 \cdot eV \cdot sec$ — **Initial longitudinal emittance, rms**

$N_{IR} := 2$ **Number of IR's**

$N_{UT} := 2$ **Number of Utility Region's**

$R_{bg} := \frac{1}{3000 \cdot hr}$ **Beam Gas Lifetime**

$u_{sep} := 10$ **Beam separation through IR, in sigmas**

CALCULATE SOME PARAMETERS

$\gamma := \dfrac{E_{ring}}{E_0}$	**Lorentz Gamma**	$\gamma = 2.664 \cdot 10^5$
$\gamma_{inj} := \dfrac{E_{inj}}{E_0}$	**Lorentz Gamma at injection**	$\gamma_{inj} = 5.329 \cdot 10^4$
$f_0 := \dfrac{c}{C} \cdot \sqrt{1 - \dfrac{1}{\gamma^2}}$	**Revolution Frequency**	$f_0 = 0.16\ kHz$
$T_0 := \dfrac{1}{f_0}$	**Revolution Period**	$T_0 = 6.231\ msec$
$B\rho := \dfrac{\sqrt{E_{ring}^{\ 2} - E_0^{\ 2}}}{c \cdot q}$	**Magnetic Rigidity**	$B\rho = \left(8.339 \cdot 10^5\right)\ tesla \cdot m$
$r_0 := \dfrac{q^2}{4 \cdot \pi \cdot \varepsilon_0 \cdot E_0}$	**Classical Radius**	$r_0 = \left(1.535 \cdot 10^{-18}\right)\ m$
$B_{spacing} := \dfrac{C}{B}$	**Bunch Spacing**	$B_{spacing} = 9.34\ m$
	$T_{spacing} := \left(B \cdot f_0\right)^{-1}$	$T_{spacing} = 31.155\ nsec$
$N_{total} := B \cdot N_0$	**Total particles in ring**	$N_{total} = 1.2 \cdot 10^{16}$
$E_{stored} := E_{ring} \cdot B \cdot N_0$	**Initial Stored Energy**	$E_{stored} = 480.653\ GJ$
		$E_{stored} = 114.879\ ton\_TNT$
		$E_{stored} = \left(1.907 \cdot 10^4\right)\ doz\_jelly\_doughnuts$

$$\Sigma_{inel} := 18.304 \cdot mb \cdot \left(\left(\frac{E_{ring}}{GeV}\right)^2\right)^{0.095} + 60.12 \cdot mb \cdot \left(\left(\frac{E_{ring}}{GeV}\right)^2\right)^{-0.34} + 32.84 \cdot mb \cdot \left(\left(\frac{E_{ring}}{GeV}\right)^2\right)^{-0.55}$$

Total Inelastic Cross-section $\Sigma_{inel} = 194.171\ mb$

LATTICE CALCULATIONS

$C = 1868\ km$ **Circumference**

$L_{IR} := 10 \cdot km$ **Length of single IR Region**

$L_{UT} := 10 \cdot km$ **Length of single Utility Region**

$N_{SA} := 10$ **Number of Service Areas per arc**

$N_{ArcReg} := 2 \cdot (N_{SA} + 1)$ **Number of "Arc Regions" per ring** $N_{ArcReg} = 22$

$L_{ArcReg} := \frac{C - (N_{IR} \cdot L_{IR} + N_{UT} \cdot L_{UT})}{N_{ArcReg}}$ **Length of "Arc Region"** $L_{ArcReg} = 83.091\ km$

$N_{halfcells} := 6000$ **Number of half cells in ring**

"Classical" number of half cells: $\sqrt{L_{ArcReg} \cdot N_{ArcReg} \cdot \frac{1}{0.3048 \cdot m}} = 2448.954$

$L_{halfcell} := L_{ArcReg} \cdot \frac{N_{ArcReg}}{N_{halfcells}}$ **Half Cell Length (distance between quads)** $L_{halfcell} = 304.667\ m$

$N_{cells} := \frac{N_{halfcells}}{2}$ **Number of Cells** $N_{cells} = 3000$

SOME IR NUMBERS:

$L_{star} := 20 \cdot m$ **Half-Length of Detector "free space" (IP-to-quad)**

$G_{trip} := 500 \cdot \frac{tesla}{m}$ **Triplet quadrupole gradient**

$$L_{eff} := L_{star} \cdot \left(1 + \frac{2 \cdot B\rho}{G_{trip} \cdot {L_{star}}^2}\right)$$

Effective distance to triplet

$$L_{eff} = 186.782\ m$$

$$\beta x_{peak} := \frac{{L_{eff}}^2}{\beta x_{star}} \qquad \beta y_{peak} := \frac{{L_{eff}}^2}{\beta y_{star}}$$

Peak amplitude function in the triplet (and in the ring)

$$\beta x_{peak} = 69.775\ km$$

$$\beta y_{peak} = 69.775\ km$$

CELL PARAMETERS, etc.

$\mu_{cell} := 90 \cdot deg$ **Phase advance per cell**

$\mu_{IR} := 8 \cdot \pi$ **Phase advance per IR**

$\mu_{UT} := 2 \cdot \pi \cdot \left(1 + \frac{.718}{2}\right)$ **Phase advance per Utility Region**

$\nu := \frac{1}{2 \cdot \pi} \cdot \left(\mu_{IR} \cdot N_{IR} + \mu_{UT} \cdot N_{UT} + \mu_{cell} \cdot N_{cells}\right)$ **tune** $\nu = 760.718$

$\xi_{nat} := -\nu - \frac{N_{IR}}{4 \cdot \pi} \cdot \frac{2 \cdot L_{eff}}{\beta x_{star}}$ **Natural Chromaticity** $\xi_{nat} = -879.627$

$\gamma_t := \nu$ **Transition Gamma** $\gamma_t = 760.718$

$F := \frac{L_{halfcell}}{2 \cdot \sin\left(\frac{\mu_{cell}}{2}\right)}$ **Cell Quad Focal Length** $F = 215.432\ m$

$GL_{quad} := \frac{B\rho}{F}$ **Integrated quad field** $GL_{quad} = 3870.877\ tesla$

if $L_{quad} := 22 \cdot m$ **, then** $G_{quad} := \frac{GL_{quad}}{L_{quad}}$ $G_{quad} = 175.949\ \frac{tesla}{m}$

$N_{dipolehalfcell} := 1$ **Number of dipoles per half cell**

$L_{dipole} := \frac{L_{halfcell} - L_{quad}}{N_{dipolehalfcell}}$ **Dipole magnet length** $L_{dipole} = 282.667\ m$

Total number of dipoles
(10 are missing from each Service Area)

(Note: neglecting Dispersion Suppressors at ends of arcs,
assumed to be 90 deg., perhaps shorter cells)

$N_{dipoles} := N_{dipolehalfcell} \cdot N_{halfcells} - 10 \cdot 2 \cdot N_{SA}$ $N_{dipoles} = 5800$

$packfrac := \frac{L_{dipole} \cdot N_{dipoles}}{C}$ **Packing Fraction** $packfrac = 0.878$

$$B_{field} := \frac{2 \cdot \pi \cdot B\rho}{packfrac \cdot C}$$

Bend Field

$$B_{field} = 3.196\ tesla$$

$$\theta_{dipole} := \frac{2 \cdot \pi}{N_{dipoles}}$$

Bend angle per dipole

$$\theta_{dipole} = 1.083\ mr$$

Bend angle per half cell

$$\theta_{halfcell} := \theta_{dipole} \cdot N_{dipolehalfcell}$$

$$\theta_{halfcell} = 1.083\ mr$$

$$\rho := \frac{B\rho}{B_{field}}$$

Bend radius

$$\rho = 260.929\ km$$

LATTICE FUNCTIONS

$$\alpha := \sqrt{\frac{1+\sin\left(\frac{\mu_{cell}}{2}\right)}{1-\sin\left(\frac{\mu_{cell}}{2}\right)}}$$

Courant-Snyder "alpha" after focusing quad $\alpha = 2.414$

$$\beta_{max} := \frac{L_{halfcell}}{\sin\left(\frac{\mu_{cell}}{2}\right)} \cdot \alpha$$

Cell maximum amplitude function $\beta_{max} = 1040.197\ m$

$$\beta_{min} := \frac{L_{halfcell}}{\sin\left(\frac{\mu_{cell}}{2}\right)} \cdot \frac{1}{\alpha}$$

Cell minimum amplitude function $\beta_{min} = 178.47\ m$

$$\beta_{ave} := \frac{\beta_{max} + \beta_{min}}{2}$$

Average amplitude function in arc (approximate) $\beta_{ave} = 609.333\ m$

$$D_{max} := \frac{\theta_{halfcell} \cdot L_{halfcell}}{\sin\left(\frac{\mu_{cell}}{2}\right)^2} \cdot \left(1 + \frac{1}{2} \cdot \sin\left(\frac{\mu_{cell}}{2}\right)\right)$$

Maximum arc Dispersion function $D_{max} = 0.893\ m$

$$D_{min} := \frac{\theta_{halfcell} \cdot L_{halfcell}}{\sin\left(\frac{\mu_{cell}}{2}\right)^2} \cdot \left(1 - \frac{1}{2} \cdot \sin\left(\frac{\mu_{cell}}{2}\right)\right)$$

Minimum arc Dispersion function $D_{min} = 0.427\ m$

$$D_{ave} := \frac{D_{max} + D_{min}}{2}$$

Average arc Dispersion function (approximate) $D_{ave} = 0.66\ m$

$$G_{dipole} := 0$$

Gradient in main dipole magnets

$$scriptD := 2 \cdot D_{ave} \cdot \frac{G_{dipole}}{B_{field}}$$

(for combined function lattices, where all bending magnets have a systematic gradient...)

(Disable Appropriate Expression)

$$scriptD := \frac{1}{{\gamma_t}^2}$$

(for separated function lattices...)

$$scriptD = 1.728 \cdot 10^{-6}$$

$$scriptH_{ave} := \frac{1}{2} \cdot \left(\frac{{D_{max}}^2}{\beta_{max}} + \frac{{D_{min}}^2}{\beta_{min}} \right)$$

For equilibrium emittance calculations

$$scriptH_{ave} = \left(8.939 \cdot 10^{-4}\right)\ m$$

SYNCHROTRON RADIATION AND DAMPING

$$I_{beam} := q \cdot f_0 \cdot B \cdot N_0$$

Beam Current

$$I_{beam} = 308.557\ mA$$

$$\Delta E_{synch\_turn} := \frac{4 \cdot \pi \cdot r_0 \cdot E_{ring}^{\ 4}}{3 \cdot \rho \cdot E_0^{\ 3}}$$

Synchrotron radiation per particle per turn

$$\Delta E_{synch\_turn} = 116.51\ MeV$$

$$w_c := \frac{9}{8 \cdot \pi} \cdot \frac{hbar \cdot c}{r_0} \cdot \left(\frac{2 \cdot \pi \cdot \rho}{C}\right) \cdot \frac{\Delta E_{synch\_turn}}{E_{ring}}$$

Critical Photon Energy

$$w_c = 18.833\ keV$$

$$\lambda_c := \frac{2 \cdot \pi \cdot hbar \cdot c}{w_c}$$

Critical Photon Wavelength

$$\lambda_c = 0.658\ Agstm$$

$$P_{synch\_tot} := f_0 \cdot \Delta E_{synch\_turn} \cdot B \cdot N_0 \cdot 2$$

Total Initial Synchrotron Power per ring (two beams)

$$P_{synch\_tot} = \left(7.19 \cdot 10^4\right)\ kW$$

$$P_{synch\_meter} := \frac{P_{synch\_tot}}{2 \cdot \pi \cdot \rho}$$

Synchrotron power per meter into the dipoles (two beams)

$$P_{synch\_meter} = 43.856\ \frac{watt}{m}$$

$$\tau_0 := \frac{E_{ring}}{\Delta E_{synch\_turn} \cdot f_0}$$

Characteristic damping time

$$\tau_0 = 3.714\ hr$$

$$\tau_x := \frac{2}{1 - scriptD} \cdot \tau_0$$

Horizontal Amplitude Damping Time

$$\tau_x = 7.428\ hr$$

$$\tau_y := 2 \cdot \tau_0$$

Vertical Amplitude Damping Time

$$\tau_y = 7.428\ hr$$

$$\tau_p := \frac{2}{2 + scriptD} \cdot \tau_0$$

Longitudinal Amplitude Damping Time

$$\tau_p = 3.714\ hr$$

$$\varepsilon_{equil} := \pi \cdot \frac{55 \cdot \sqrt{3}}{2^4 \cdot 3^2} \cdot \left(\frac{scriptH_{ave}}{1 - scriptD}\right) \cdot \frac{w_c}{E_0}$$

Equilibrium (ideal) Transverse Emittance

$$\varepsilon_{equil} = 0.012\ \pi \cdot mm \cdot mr$$

$$\sigma_{p\_equil} := \sqrt{\frac{55 \cdot \sqrt{3}}{2^4 \cdot 3^2} \cdot \left(\frac{1}{2 + scriptD}\right) \cdot \frac{w_c}{E_{ring}}}$$

Equilibrium (ideal) Momentum Spread

$$\sigma_{p\_equil} = 4.992 \cdot 10^{-6}$$

RF PARAMETERS

Initial longitudinal emittance (rms) at storage: $S_0 = 0.8\ eV \cdot sec$

$h := 8 \cdot B$ **Harmonic Number** $h = 1.6 \cdot 10^6$

$f_{rf} := h \cdot f_0$ **RF frequency** $f_{rf} = 256.782\ MHz$

RF Voltage at storage $eV_{rf} := 120 \cdot MeV$

Synchronous phase at storage $\phi_s := 180 \cdot deg$

Slip factor at storage $\eta := \frac{1}{{\gamma_t}^2} - \frac{1}{\gamma^2}$ $\eta = 0.002\ 10^{-3}$

Bucket Area at Storage: $A_{bucket} := \frac{16}{2 \cdot \pi \cdot f_{rf}} \cdot \sqrt{\frac{eV_{rf} \cdot E_{ring}}{2 \cdot \pi \cdot h \cdot |\eta|}}$ $A_{bucket} = 412.109\ eV \cdot sec$

Synchrotron Tune at Storage:

$\nu_s := \sqrt{-\frac{h \cdot \eta \cdot eV_{rf} \cdot \cos(\phi_s)}{2 \cdot \pi \cdot E_{ring}}}$ $\nu_s = 4.596 \cdot 10^{-4}$ $\frac{1}{\nu_s} = 2175.879$

Synchrotron Frequency: $f_s := f_0 \cdot \nu_s$ $f_s = 0.074\ Hz$

Initial bunch length (rms) at storage $\sigma_{t_0} := \sqrt{\frac{S_0}{\pi \cdot 2 \cdot \pi \cdot h \cdot f_0}} \cdot \left(\frac{2 \cdot \pi \cdot h \cdot |\eta|}{eV_{rf} \cdot E_{ring} \cdot |\cos(\phi_s)|}\right)^{\frac{1}{4}}$

$\sigma_{t_0} = 0.062\ nsec$ $\sigma_{s_0} := \sigma_{t_0} \cdot c$ $\sigma_{s_0} = 1.848\ cm$

Initial relative momentum spread (rms) at storage $\sigma_{p_0} := \left(\frac{2 \cdot eV_{rf} \cdot |\cos(\phi_s)| \cdot (S_0)^2 \cdot {f_{rf}}^2}{\pi \cdot h \cdot |\eta| \cdot {E_{ring}}^3}\right)^{\frac{1}{4}}$

$\sigma_{p_0} = 1.653 \cdot 10^{-5}$

$$I_{beam} = 308.557\ mA$$

$$I_{bunch} := \frac{I_{beam}}{B}$$

Bunch current

$$I_{bunch} = 1.543\ \mu A$$

$$I_{peak} := I_{bunch} \cdot \frac{C}{\sqrt{2 \cdot \pi} \cdot \sigma_{s_0}}$$

Peak current

$$I_{peak} = 62.229\ amp$$

ACCELERATION PARAMETERS **(assume blow up long. emittance after injection)**

$E_{inj} = 50\ TeV$ $f_{rf} = 256.782\ MHz$ $S_{inj} := \frac{S_0}{2}$ $S_{inj} = 0.4\ eV \cdot sec$

$\eta_{inj} := \frac{1}{\gamma_t^2} - \frac{1}{\gamma_{inj}^2}$ **Slip factor at injection** $\eta_{inj} = 0.002\ 10^{-3}$

RF Voltage for acceleration: $eV_{inj} := 250 \cdot MeV$

Bucket Area at injection:

$$A_{bucket\_inj} := \frac{16}{2 \cdot \pi \cdot f_{rf}} \cdot \sqrt{\frac{eV_{inj} \cdot E_{inj}}{2 \cdot \pi \cdot h \cdot |\eta_{inj}|}} \qquad A_{bucket\_inj} = 266.041\ eV \cdot sec$$

Synchrotron Tune at injection:

$$\nu_{s\_inj} := \sqrt{-\frac{h \cdot \eta_{inj} \cdot eV_{inj} \cdot \cos(\phi_s)}{2 \cdot \pi \cdot E_{inj}}} \qquad \nu_{s\_inj} = 0.001$$

$$\frac{1}{\nu_{s\_inj}} = 674.237$$

Synchrotron Frequency: $f_{s\_inj} := f_0 \cdot \nu_{s\_inj}$ $f_{s\_inj} = 0.238\ Hz$

Bunch length (rms) at injection

$$\sigma_{t\_inj} := \sqrt{\frac{S_{inj}}{\pi \cdot 2 \cdot \pi \cdot h \cdot f_0}} \cdot \left(\frac{2 \cdot \pi \cdot h \cdot |\eta_{inj}|}{eV_{inj} \cdot E_{inj} \cdot |\cos(\phi_s)|}\right)^{\frac{1}{4}}$$

$\sigma_{t\_inj} = 0.054\ nsec$ $\sigma_{s\_inj} := \sigma_{t\_inj} \cdot c$ $\sigma_{s\_inj} = 1.626\ cm$

Relative momentum spread (rms) at injection

$$\sigma_{p\_inj} := \left(\frac{2 \cdot eV_{inj} \cdot |\cos(\phi_s)| \cdot (S_{inj})^2 \cdot f_{rf}^2}{\pi \cdot h \cdot |\eta_{inj}| \cdot E_{inj}^3}\right)^{\frac{1}{4}}$$

$$\sigma_{p\_inj} = 4.695 \cdot 10^{-5}$$

Synchronous phase for acceleration $\phi_{acc} := 150 \cdot deg$

Acceleration Time

$$T_{acc} := \frac{E_{ring} - E_{inj}}{eV_{inj} \cdot \sin\left(\phi_{acc}\right) \cdot f_0}$$

$$T_{acc} = 166.159\ min$$

BEAM SIZE AND APERTURE

Beam +
(arc) at beginning of store

$$\sigma_{x_0} := \sqrt{\frac{\varepsilon x_{n_0} \cdot \beta_{max}}{\pi \cdot \gamma} + \left(D_{max} \cdot \sigma_{p_0}\right)^2}$$

$$\sigma_{x_0} = 0.09\ mm$$

Beam size at interaction point

$$\sigma x_{star} := \sqrt{\frac{\varepsilon x_{n_0} \cdot \beta x_{star}}{\pi \cdot \gamma}}$$

$$\sigma x_{star} = 1.937\ \mu m$$

Beam size at interaction point

$$\sigma y_{star} := \sqrt{\frac{\varepsilon y_{n_0} \cdot \beta y_{star}}{\pi \cdot \gamma}}$$

$$\sigma y_{star} = 1.937\ \mu m$$

Beam size in triplet quads

$$\sigma x_{peak} := \sqrt{\frac{\varepsilon x_{n_0} \cdot \beta x_{peak}}{\pi \cdot \gamma}}$$

$$\sigma x_{peak} = 0.724\ mm$$

Beam size in triplet quads

$$\sigma y_{peak} := \sqrt{\frac{\varepsilon y_{n_0} \cdot \beta y_{peak}}{\pi \cdot \gamma}}$$

$$\sigma y_{peak} = 0.724\ mm$$

Beam size (arc) at injection

$$\sigma_{x\_inj} := \sqrt{\frac{\varepsilon x_{n_0} \cdot \beta_{max}}{\pi \cdot \gamma_{inj}} + \left(D_{max} \cdot \sigma_{p\_inj}\right)^2}$$

$$\sigma_{x\_inj} = 0.202\ mm$$

Presume some injection IR optics...

$$\beta_{star\_inj} := 4 \cdot m$$

$$\beta_{peak\_inj} := \frac{{L_{eff}}^2}{\beta_{star\_inj}}$$

$$\beta_{peak\_inj} = 8721.883\ m$$

Then, the beam size in IR at injection will be:

$$\sigma_{star\_inj} := \sqrt{\frac{\varepsilon x_{n_0} \cdot \beta_{star\_inj}}{\pi \cdot \gamma_{inj}}}$$

Beam size at interaction point

$$\sigma_{star\_inj} = 12.252\ \mu m$$

$$\sigma_{peak\_inj} := \sqrt{\frac{\varepsilon x_{n_0} \cdot \beta_{peak\_inj}}{\pi \cdot \gamma_{inj}}}$$

Beam size in triplet quads

$$\sigma_{peak\_inj} = 0.572\ mm$$

Take beampipe radius to be: $b_{pipe} := 16.5 \cdot mm$

check: $10 \cdot \sigma_{x\_inj} = 2.02\ mm$ $\quad 10 \cdot \sigma x_{peak} = 7.237\ mm$

Minimum Triplet Aperture Requirement at Injection $\quad E_{inj} = 50\ TeV$

$$r_{trip} := \sigma_{peak\_inj} \cdot \left(7.5 + \frac{u_{sep}}{2}\right) \cdot 1.5 \qquad r_{trip} = 10.728\ mm$$

VERTICAL CROSSING ANGLE at INTERACTION POINT

$\sigma_{s_0} = 1.848\ cm$ $\sigma y_{star} = 1.937\ \mu m$ **Beam Separation distance (sigmas):** $u_{sep} = 10$

$\beta y_{star} = 50\ cm$

Crossing angle: $\alpha_{cross_0} := \sqrt{\dfrac{(u_{sep})^2 \cdot \varepsilon y_{n_0}}{\pi \cdot \gamma \cdot \beta y_{star}}}$

Luminosity reduction factor:

$$\frac{1}{\sqrt{1 + \left(\alpha_{cross_0}\right)^2 \cdot \left(\dfrac{\left(\sigma_{s_0}\right)^2 \cdot \pi \cdot \gamma}{4 \cdot \beta y_{star} \cdot \varepsilon y_{n_0}}\right)}} = 98.336\ 1\%$$

$\alpha_{cross_0} = 38.746\ \mu r$

Length of luminous region (rms):

$$\Sigma_{lum_0} := \frac{\sigma_{s_0}}{\sqrt{2}} \cdot \frac{1}{\sqrt{1 + \left(\alpha_{cross_0}\right)^2 \cdot \left(\dfrac{\left(\sigma_{s_0}\right)^2 \cdot \pi \cdot \gamma}{4 \cdot \beta y_{star} \cdot \varepsilon y_{n_0}}\right)}}$$

$\Sigma_{lum_0} = 12.847\ mm$

LONG RANGE TUNE SHIFT

Distance from IP to Separation Dipoles: $L_{sepdip} := 50 \cdot m$

Number of long range interactions: $n_{LR} := \dfrac{2 \cdot L_{sepdip}}{\dfrac{B_{spacing}}{2}}$ $n_{LR} = 21.413$

Total LR Tune Shift:

$$\Delta\nu_{LR_0} := \frac{2 \cdot N_0 \cdot L_{sepdip} \cdot r_0}{\pi \cdot \gamma \cdot \left(\alpha_{cross_0}\right)^2 \cdot \beta y_{star} \cdot B_{spacing}} \cdot N_{IR}$$

$\Delta\nu_{LR_0} = 3.138\ 10^{-3}$

Initial head-on tune shift:

$$\Delta\nu_{ho_0} := \frac{r_0 \cdot N_0}{4 \cdot \varepsilon y_{n_0}} \cdot N_{IR}$$

$\Delta\nu_{ho_0} = 7.328\ 10^{-3}$

figure of merit: $\Delta\nu_{LR_0} + \Delta\nu_{ho_0} = 0.01$

Maximum displacement within the triplet quadrupoles

$$\Delta y_{max} := \frac{\alpha_{cross_0}}{2} \cdot \sqrt{\beta y_{peak} \cdot \beta y_{star}} \qquad \Delta y_{max} = 3.619\ mm$$

check maximum separation: $\frac{2 \cdot \Delta y_{max}}{\sigma y_{peak}} = 10$

Minimum Triplet Aperture Requirement at Storage:

$$rx_{trip} := \left(7 \cdot \sigma x_{peak}\right) \cdot 1.5 \qquad rx_{trip} = 7.599\ mm$$

$$ry_{trip} := \left(\Delta y_{max} + 7 \cdot \sigma y_{peak}\right) \cdot 1.5 \qquad ry_{trip} = 13.027\ mm$$

$$b_{pipe} = 16.5\ mm$$

LUMINOSITY AND BEAM PARAMETER TIME EVOLUTION

Time increment $\Delta t := 0.1 \cdot hr$ **Time Steps:** $i := 0, 1 .. 200$ $t_i := i \cdot \Delta t$

Initial Luminosity

$$L_0 := \frac{B \cdot f_0 \cdot \gamma}{4 \cdot \sqrt{\varepsilon x_{n_0} \cdot \varepsilon y_{n_0} \cdot \beta x_{star} \cdot \beta y_{star}}} \cdot \left(N_0\right)^2 \cdot \frac{1}{\sqrt{1 + \left(\alpha_{cross_0}\right)^2 \cdot \left(\frac{\left(\sigma_{s_0}\right)^2 \cdot \pi \cdot \gamma}{4 \cdot \beta y_{star} \cdot \varepsilon y_{n_0}}\right)}}$$

$$L_0 = \left(2.409 \cdot 10^{35}\right)\ cm^{-2} \cdot sec^{-1}$$

Emittance growth rate due to quantum emissions from Synchrotron Radiation:

$$\varepsilon_{emission} := \pi \cdot \left(\frac{scriptH_{ave}}{\tau_0 \cdot E_0}\right) \cdot \left(\frac{55 \cdot \sqrt{3}}{2^4 \cdot 3^2}\right) \cdot w_c$$

$$\varepsilon_{emission} = \left(8.877 \cdot 10^{-7}\right) \frac{\pi \cdot mm \cdot mr}{sec}$$

Introduce vertical coupling term from horizontal quantum emission i.e. ratio of equilibrium emittances

$$r_e := 0.1$$

INTRABEAM SCATTERING EMITTANCE GROWTH PARAMETERS

Parameterization per Wei (PAC93, p.3651)

Assume transverse emittance the same in both planes starting out

IBS Transverse Scaling Factor 1.0 = 100%, 0.5 = 50% etc... $IBST := 1.0$

IBS Longitudinal Scaling Factor $IBSL := 1.0$

n = 1 non-coupled, n=2 fully coupled $n_c := 2$

Coulomb Logarithm $L_c := 20$

Initial value of partition fuction

$$d_0 := \sqrt{\frac{1}{\frac{\beta_{ave} \cdot \varepsilon x_{n_0}}{\pi \cdot \gamma \cdot \left(D_{ave}{}^2 \cdot \left(\sigma_{p_0}\right)^2\right)} + 1}}$$

$$d_0 = 0.159$$

Initial Longitudinal IBS Growth Time

$$\tau p_0 := \left(\frac{(\pi)^3}{16 \cdot \gamma_t} \cdot N_0 \cdot \frac{r_0 \cdot L_c}{\left(\varepsilon x_{n_0}\right)^2} \cdot \frac{r_0 \cdot E_0}{S_0} \cdot \left(\frac{1 - \left(d_0\right)^2}{d_0} \right) \right)^{-1}$$

$$\tau p_0 = 212.204\ hr$$

Initial Transverse IBS Growth Time

$$\tau x_0 := \left(\left(\frac{\pi^3}{16 \cdot \gamma_t} \right) \cdot N_0 \cdot \frac{r_0 \cdot L_c}{\left(\varepsilon x_{n_0}\right)^2} \cdot \frac{r_0 \cdot E_0}{S_0} \cdot \left(\frac{d_0}{n_c} \right) \right)^{-1}$$

$$\tau x_0 = \left(1.631 \cdot 10^4\right)\ hr$$

Include heating of longitudinal emittance during store:

$$S_{dot} := .4 \cdot \frac{eV \cdot sec}{hr}$$

$$\sigma p_{dot}(S) := \frac{1}{2} \cdot \left(\frac{2 \cdot eV_{rf} \cdot \left|\cos\left(\phi_s\right)\right| \cdot f_{rf}^{\ 2}}{\pi \cdot h \cdot |\eta| \cdot E_{ring}^{\ 3} \cdot (S)^2} \right)^{\frac{1}{4}} \cdot S_{dot}$$

PARAMETER EVOLUTION DURING THE STORE:

Vary crossing angle to keep beam separation constant...

$$\begin{bmatrix} L_{i+1} \\ N_{i+1} \\ \varepsilon x_{n_{i+1}} \\ \varepsilon y_{n_{i+1}} \\ \tau p_{i+1} \\ \tau x_{i+1} \\ S_{i+1} \\ \sigma_{p_{i+1}} \\ \sigma_{s_{i+1}} \\ d_{i+1} \\ \alpha_{cross_{i+1}} \end{bmatrix} := \begin{bmatrix} \dfrac{B \cdot f_0 \cdot \gamma}{4 \cdot \sqrt{\varepsilon x_{n_i} \cdot \varepsilon y_{n_i} \cdot \beta x_{star} \cdot \beta y_{star}}} \cdot \left(N_i\right)^2 \cdot \dfrac{1}{\sqrt{1 + \left(\alpha_{cross_i}\right)^2 \cdot \left(\dfrac{\left(\sigma_{s_i}\right)^2 \cdot \pi \cdot \gamma}{4 \cdot \beta y_{star} \cdot \varepsilon y_{n_i}}\right)}} \\ N_i - \left(\dfrac{N_{IR} \cdot L_i \cdot \Sigma_{inel}}{B} + N_i \cdot R_{bg}\right) \cdot \Delta t \\ \varepsilon x_{n_i} \cdot \left(1 - \left(\dfrac{2}{\tau_x} - \dfrac{2}{\tau x_i}\right) \cdot \Delta t\right) + \varepsilon_{emission} \cdot \left(1 - r_e\right) \cdot \Delta t \\ \varepsilon y_{n_i} \cdot \left(1 - \left(\dfrac{2}{\tau_y} - \dfrac{2}{\tau x_i}\right) \cdot \Delta t\right) + r_e \cdot \varepsilon_{emission} \cdot \Delta t \\ \left(\left(\dfrac{\pi^3}{16 \cdot \gamma_t}\right) \cdot N_i \cdot L_c \cdot \dfrac{r_0}{\left(\varepsilon x_{n_i} \cdot \varepsilon y_{n_i}\right)} \cdot \dfrac{r_0 \cdot E_0}{S_i} \cdot \left(\dfrac{1 - \left(d_i\right)^2}{d_i}\right) \cdot IBST\right)^{-1} \\ \left(\left(\dfrac{\pi^3}{16 \cdot \gamma_t}\right) \cdot N_i \cdot L_c \cdot \dfrac{r_0}{\left(\varepsilon x_{n_i} \cdot \varepsilon y_{n_i}\right)} \cdot \dfrac{r_0 \cdot E_0}{S_i} \cdot \left(\dfrac{d_i}{n_c}\right) \cdot IBST\right)^{-1} \\ S_i \cdot \left(1 + \left(\dfrac{2}{\tau p_i} \cdot IBSL - \dfrac{2}{\tau_p}\right) \cdot \Delta t\right) + S_{dot} \cdot \Delta t \\ \sigma_{p_i} \cdot \left(1 + \left(\dfrac{1}{\tau p_i} \cdot IBSL - \dfrac{1}{\tau_p}\right) \cdot \Delta t\right) + \sigma p_{dot}\left(S_i\right) \cdot \Delta t \\ c \cdot \sqrt{\dfrac{S_i}{\pi \cdot 2 \cdot \pi \cdot h \cdot f_0}} \cdot \left(\dfrac{2 \cdot \pi \cdot h \cdot |\eta|}{eV_{rf} \cdot E_{ring} \cdot \left|\cos\left(\phi_s\right)\right|}\right)^{\frac{1}{4}} \\ \sqrt{\dfrac{1}{\dfrac{\beta_{ave} \cdot \varepsilon x_{n_i}}{\pi \cdot \gamma \cdot D_{ave}{}^2 \cdot \left(\sigma_{p_i}\right)^2} + 1}} \\ \sqrt{\dfrac{u_{sep}{}^2 \cdot \varepsilon y_{n_i}}{\pi \cdot \gamma \cdot \beta y_{star}}} \end{bmatrix}$$

Transverse bunch density (arbitary units)

$$TBD := \overrightarrow{\left(\frac{N \cdot \pi \cdot mm \cdot mr}{\varepsilon x_n \cdot 10^{10}}\right)}$$

Longitudinal bunch density (arbitary units)

$$LBD := \overrightarrow{\left(\frac{N \cdot eV \cdot sec}{S \cdot 10^{10}}\right)}$$

Bunch length (time)

$$\sigma_{t_i} := \frac{\sigma_{s_i}}{c}$$

Integrated Luminosity:

$$\Sigma L_0 := 0 \cdot pb^{-1} \qquad \Sigma L_{i+1} := \Sigma L_i + L_i \cdot \Delta t$$

$$t_{100} = 10\ hr \qquad \Sigma L_{100} = \left(1.8 \cdot 10^4\right)\ pb^{-1}$$

$$t_{200} = 20\ hr \qquad \Sigma L_{200} = \left(2.846 \cdot 10^4\right)\ pb^{-1}$$

$$\max(L) = \left(6.403 \cdot 10^{35}\right) \frac{1}{s} \cdot cm^{-2}$$

Head-on tune shift:

$$\Delta\nu x_{ho_i} := \frac{r_0 \cdot N_i}{2 \cdot \varepsilon x_{n_i}} \cdot \frac{1}{1+\sqrt{\dfrac{\beta y_{star} \cdot \varepsilon y_{n_i}}{\beta x_{star} \cdot \varepsilon x_{n_i}}}} \cdot N_{IR}$$

$$\Delta\nu y_{ho_i} := \frac{r_0 \cdot N_i}{2 \cdot \varepsilon y_{n_i}} \cdot \frac{1}{1+\sqrt{\dfrac{\beta x_{star} \cdot \varepsilon x_{n_i}}{\beta y_{star} \cdot \varepsilon y_{n_i}}}} \cdot N_{IR}$$

Long-range tune shift:

$$\Delta\nu_{LR_i} := \frac{2 \cdot N_i \cdot L_{sepdip} \cdot r_0}{\pi \cdot \gamma \cdot \left(\alpha_{cross_i}\right)^2 \cdot \beta y_{star} \cdot B_{spacing}} \cdot N_{IR}$$

$\max\left(\Delta\nu x_{ho}\right) = 0.045$ $\max\left(\Delta\nu y_{ho}\right) = 0.049$ $\max\left(\Delta\nu_{LR}\right) = 0.021$

Interactions per crossing: $R_{int_i} := \dfrac{L_i \cdot \Sigma_{inel}}{B \cdot f_0}$ $R_{int_0} = 1457.466$ $\max\left(R_{int}\right) = 3873.191$

Luminous Region:

$$\Sigma_{lum_i} := \frac{\sigma_{s_i}}{\sqrt{2}} \cdot \frac{1}{\sqrt{1+\left(\alpha_{cross_i}\right)^2 \cdot \left(\dfrac{\left(\sigma_{s_i}\right)^2 \cdot \pi \cdot \gamma}{4 \cdot \beta y_{star} \cdot \varepsilon y_{n_i}}\right)}}$$

Peak number of interactions per mm per crossing at center of luminous region:

$$dR_{int_i} := \frac{R_{int_i}}{\sqrt{2 \cdot \pi} \cdot \Sigma_{lum_i}}$$

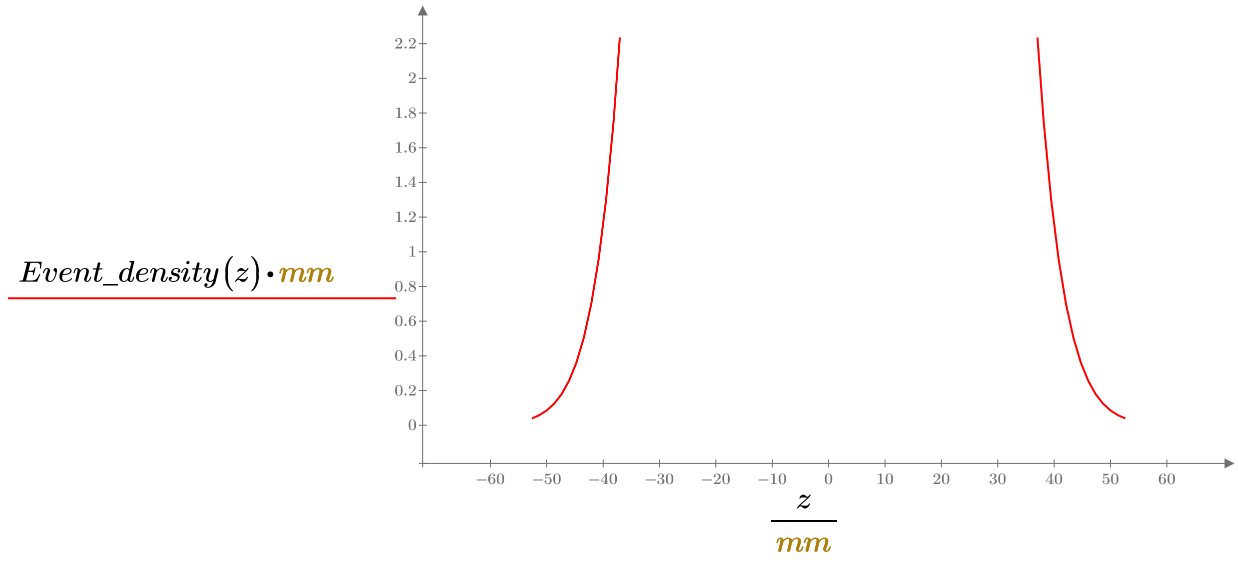
$tt_i := i$

$i\_Rmax := \mathrm{reverse}\left(\mathrm{csort}\left(\mathrm{augment}\left(tt, dR_{int} \cdot mm\right), 1\right)\right)_{0,0}$ $\max\left(dR_{int}\right) = 117.123\ mm^{-1}$

$$Event\_density(z) := \frac{R_{int_{i\_Rmax}}}{\sqrt{2 \cdot \pi} \cdot \Sigma_{lum_{i\_Rmax}}} \cdot e^{\frac{-(z)^2}{2 \cdot \left(\Sigma_{lum_{i\_Rmax}}\right)^2}}$$

$t_{i\_Rmax} = 7.4\ hr$

$z := -4 \cdot \Sigma_{lum_{i\_Rmax}}, -3.9 \cdot \Sigma_{lum_{i\_Rmax}} .. 4 \cdot \Sigma_{lum_{i\_Rmax}}$

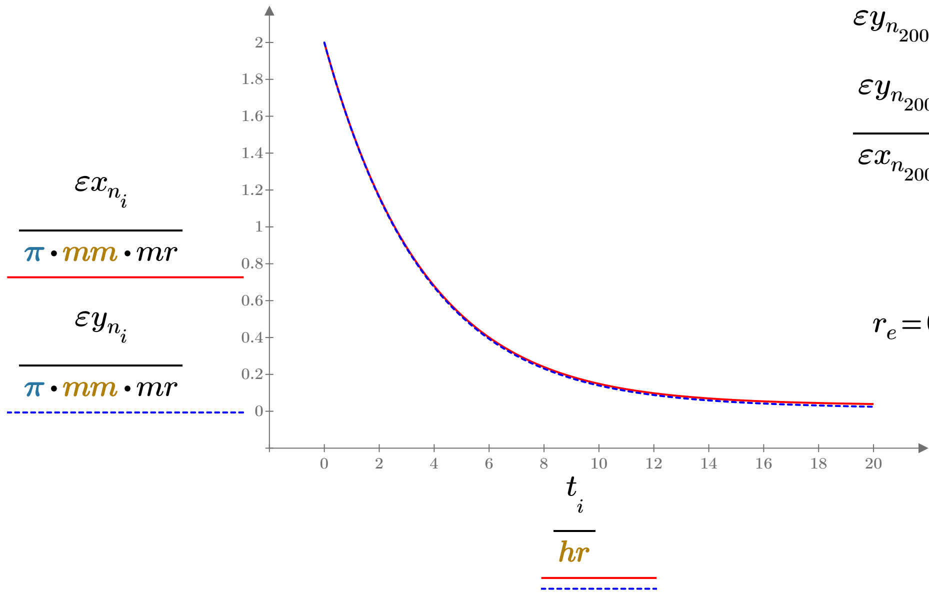

Event_density(z)·mm
2.2
2
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
−60
−50
−40
−30
−20
−10
0
10
20
30
40
50
60
z
mm

Transverse emittance

$\varepsilon x_{n_{200}} = 0.039\ \pi \cdot mm \cdot mr$

$\varepsilon y_{n_{200}} = 0.025\ \pi \cdot mm \cdot mr$

$\dfrac{\varepsilon y_{n_{200}}}{\varepsilon x_{n_{200}}} = 0.626$



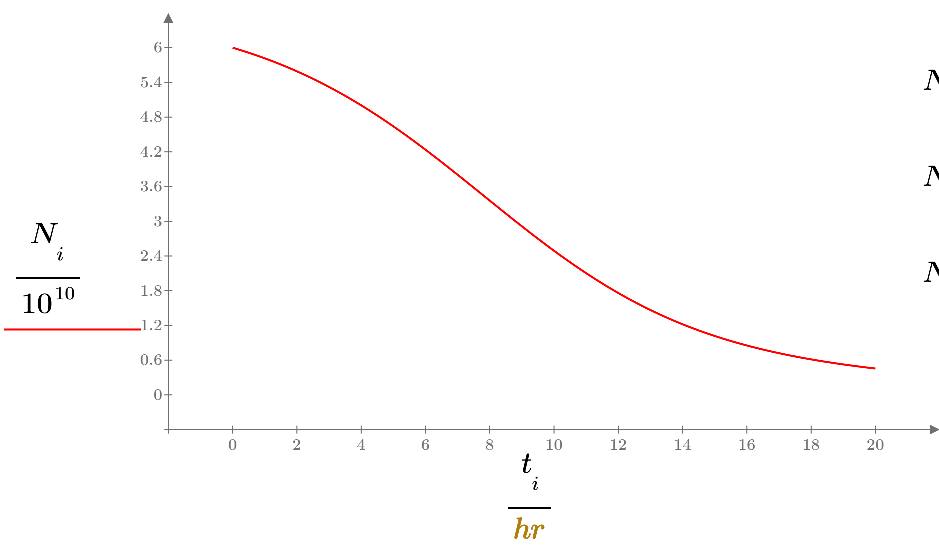
$r_e = 0.1$

Bunch Intensity

$N_0 = 6 \cdot 10^{10}$

$N_{80} = 3.357 \cdot 10^{10}$

$N_{200} = 4.553 \cdot 10^{9}$

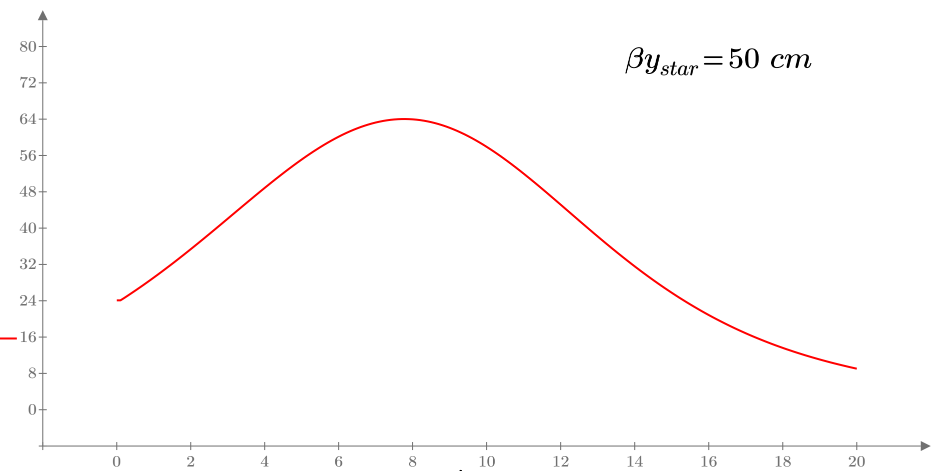


Instantaneous Luminosity

$$\max(L) = \left(6.403 \cdot 10^{35}\right)\ cm^{-2} \cdot sec^{-1}$$

$$\beta x_{star} = 50\ cm$$

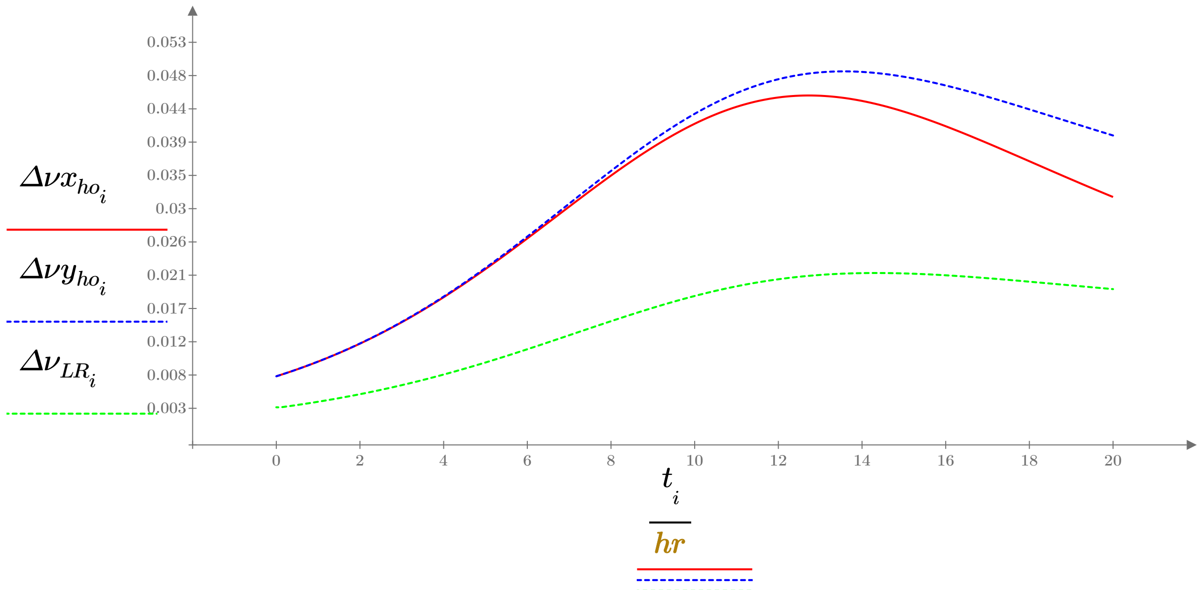
$$\beta y_{star} = 50\ cm$$



$$\frac{L_i \cdot cm^2 \cdot s}{10^{34}}$$

$$\frac{t_i}{hr}$$

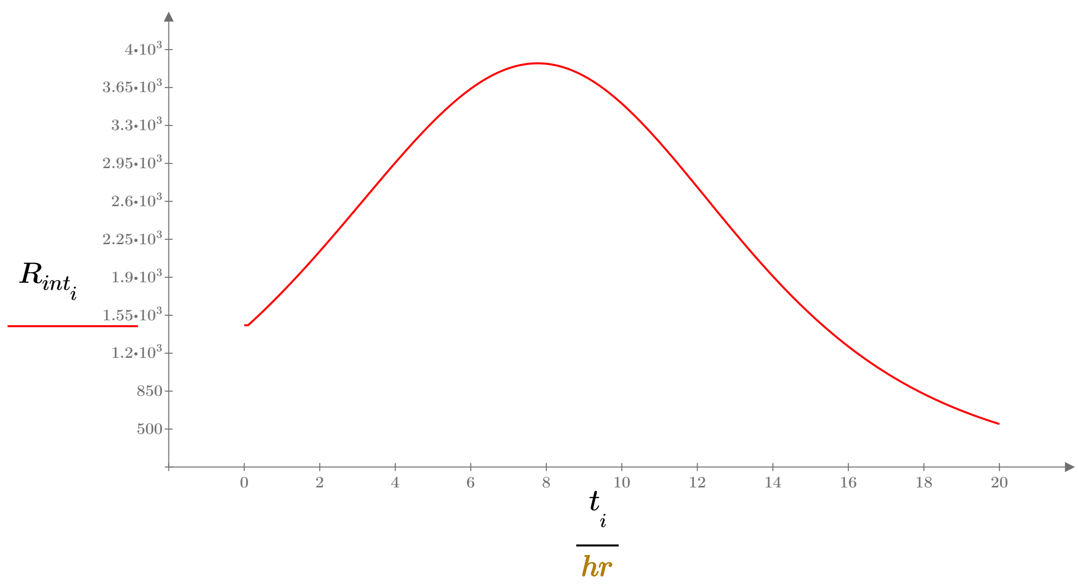
Total Head-on and Long Range Tune Shifts



$\max\left(R_{int}\right) = 3873.191$

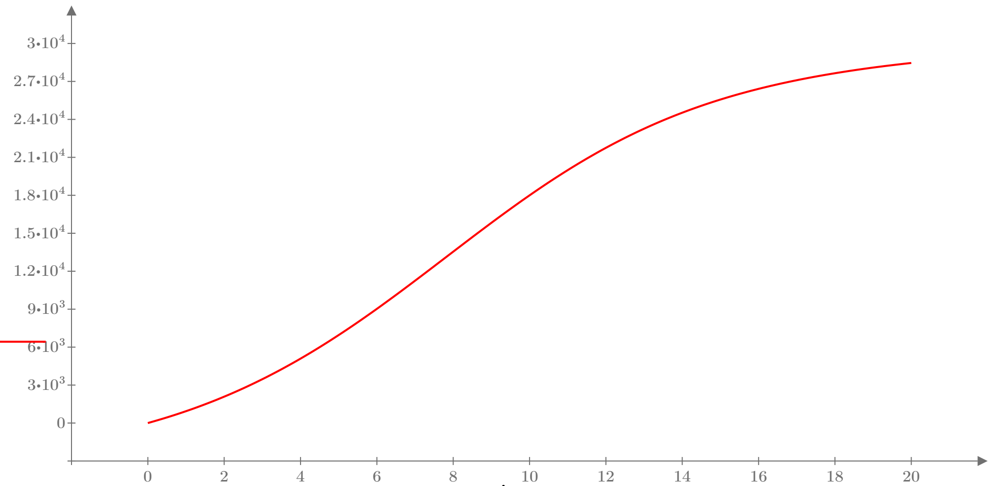
Interactions per bunch crossing

$\max\left(dR_{int}\right) = 117.123\ mm^{-1}$



$$\Sigma L_{100} = \left(1.8 \cdot 10^{4}\right)\ pb^{-1}$$

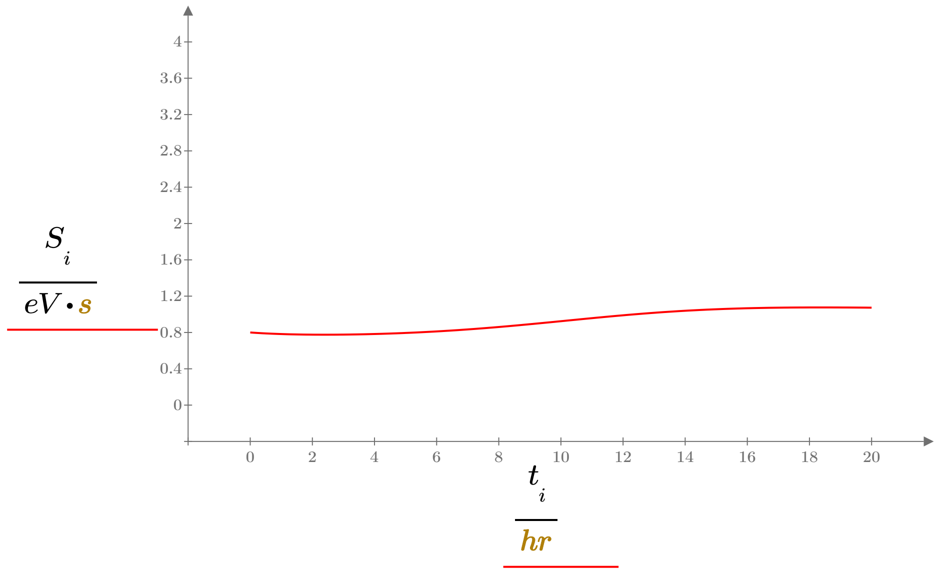
Integrated Luminosity



$\frac{\Sigma L_i}{pb^{-1}}$

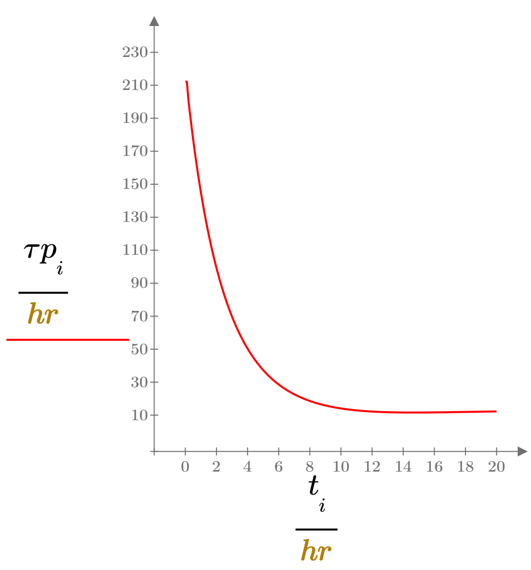
$\frac{t_i}{hr}$

Longitudinal Emittance

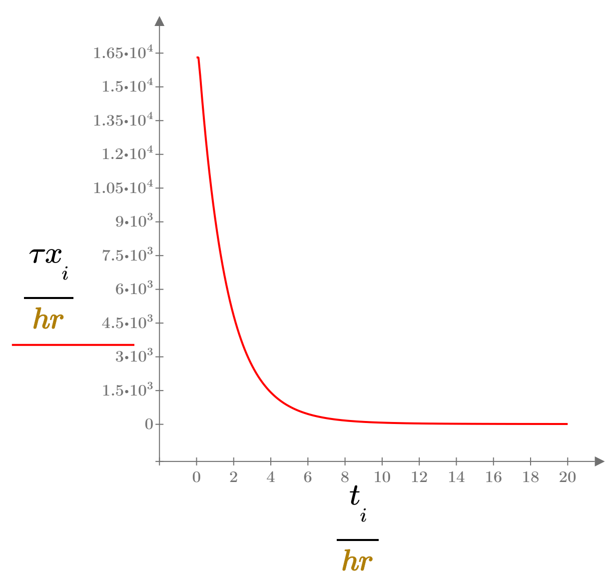


IBS GROWTH TIMES

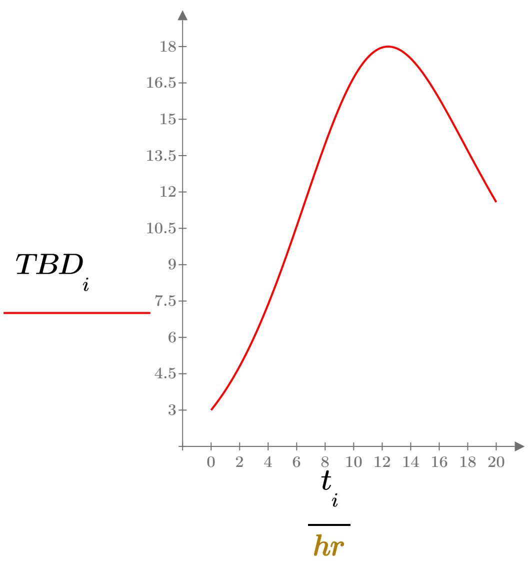
Longitudinal



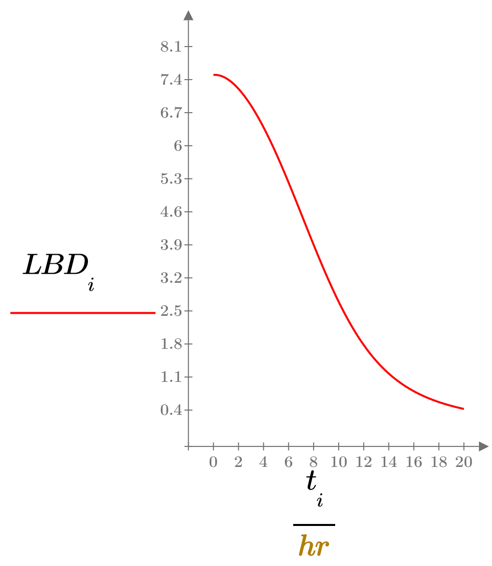
Transverse



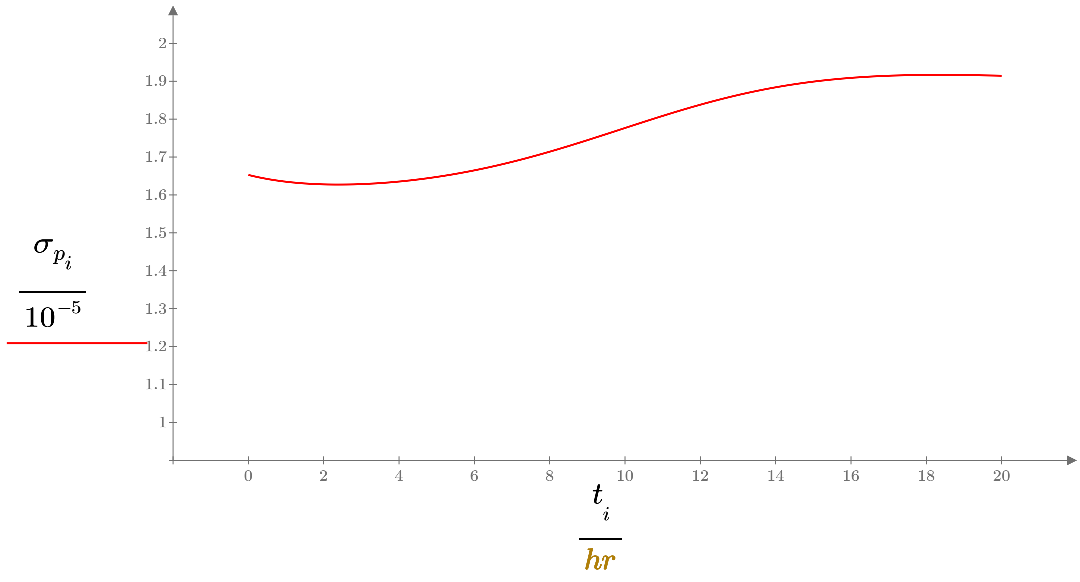
Transverse Bunch Density



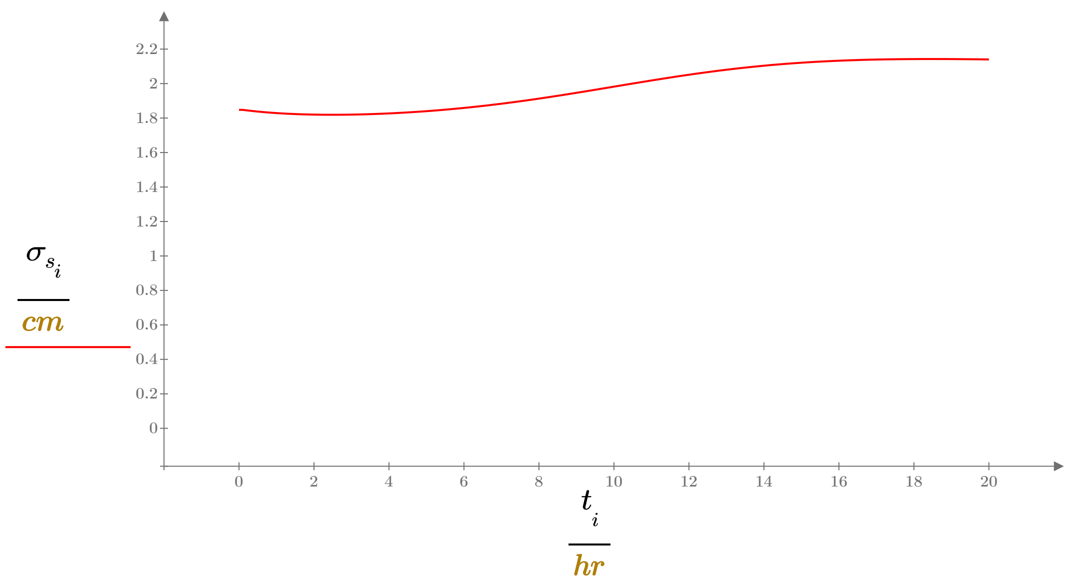
Longitudinal Bunch Density



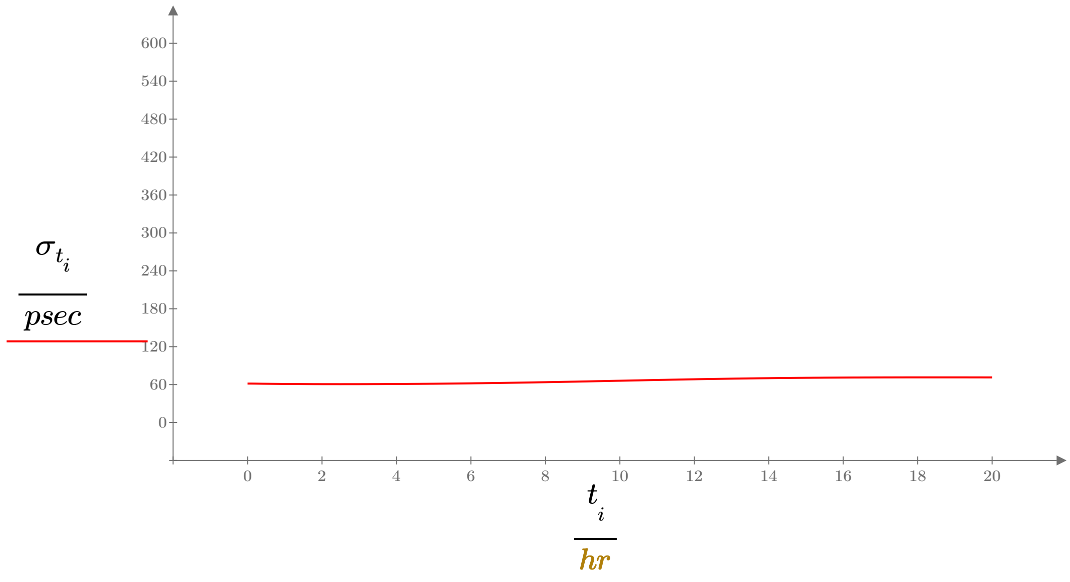
Momentum Spread



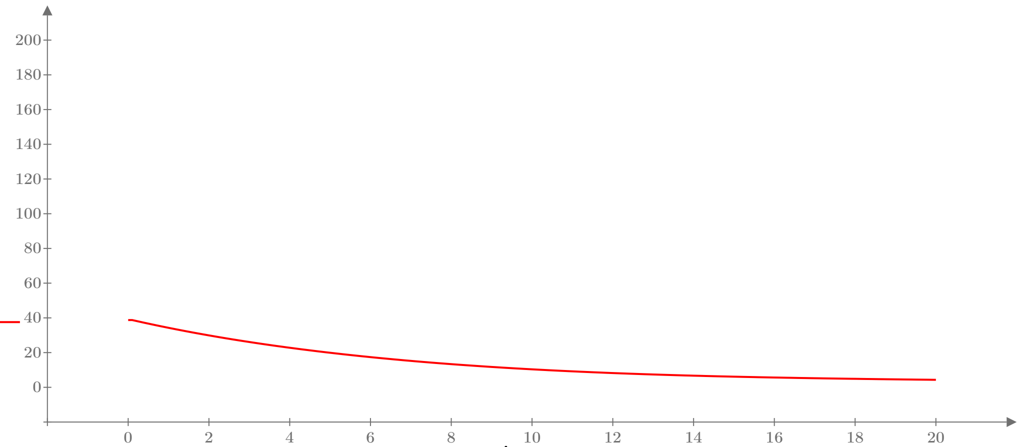
Bunch Length



Bunch Length



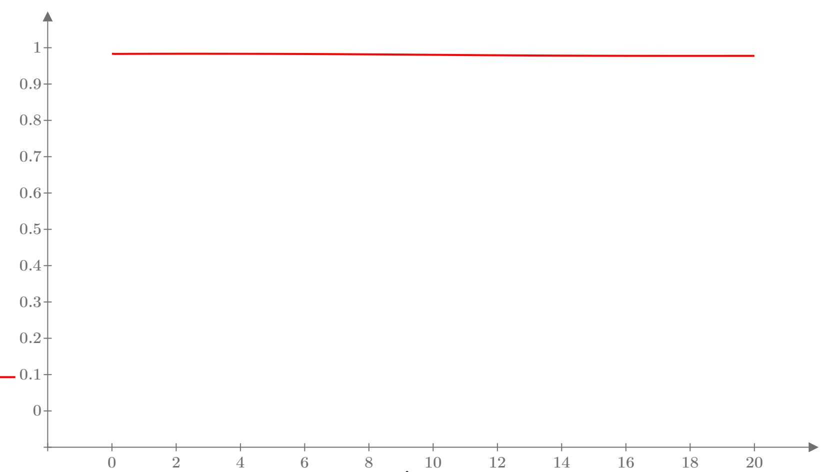
Crossing angle



$$\frac{\alpha_{cross_i}}{\mu r}$$

$$\frac{t_i}{hr}$$

Luminosity Reduction Factor



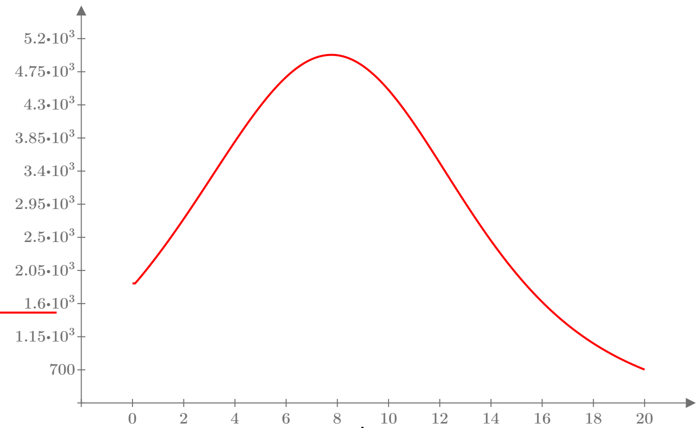
$$\frac{1}{\sqrt{1+\left(\alpha_{cross_i}\right)^2 \cdot \left(\frac{\left(\sigma_{s_i}\right)^2 \cdot \pi \cdot \gamma}{4 \cdot \beta y_{star} \cdot \varepsilon y_{n_i}}\right)}}$$

$$\frac{t_i}{hr}$$

Power delivered toward Triplet Quadrupoles from the IP:

$$P_{IP_i} := L_i \cdot \Sigma_{inel} \cdot E_{ring}$$

$$\max\left(P_{IP}\right) = 4979.587\ kwatt$$



$$\frac{P_{IP_i}}{kwatt}$$

$$\frac{t_i}{hr}$$

INSTABILITY THRESHOLDS

$\rho_{res} := 5.55 \cdot 10^{-10} \cdot ohm \cdot m$ **(Cu at 4K)**

$\delta\nu := \nu - \text{floor}(\nu)$ $\delta\nu = 0.718$

$Z_0 := \mu_0 \cdot c$ $Z_0 = 376.73\ ohm$ $b_{pipe} = 16.5\ mm$

Low Frequency:

Transverse Resistive Wall Impedance:

$$Z_{T\_rw\_lo} := \frac{C \cdot Z_0}{2 \cdot \pi \cdot b_{pipe}^{3}} \cdot \sqrt{\frac{2 \cdot \rho_{res}}{\mu_0 \cdot 2 \cdot \pi \cdot f_0 \cdot \delta\nu}}$$

$$Z_{T\_rw\_lo} = \left(2.754 \cdot 10^{4}\right) \frac{Mohm}{m}$$

Transverse Resistive Wall Growth Time at Injection (turns):

$$\tau_{T\_rw\_lo} := \frac{2 \cdot b_{pipe}^{3} \cdot E_{inj}}{q \cdot \beta_{ave} \cdot I_{beam}} \cdot \sqrt{\frac{\delta\nu \cdot (2 \cdot \pi)^3}{2 \cdot c \cdot \mu_0 \cdot \rho_{res} \cdot C^3}}$$

$$\tau_{T\_rw\_lo} = 19.313$$

High Frequency:

Transverse Resistive Wall Impedance:

$$Z_{T\_rw\_hi} := \frac{C \cdot Z_0}{2 \cdot \pi \cdot b_{pipe}^{3}} \cdot \sqrt{\frac{2 \cdot \rho_{res} \cdot \sigma_{s_0}}{\mu_0 \cdot c}}$$

$$Z_{T\_rw\_hi} = 5.817\ \frac{Mohm}{m}$$

Longitudinal Microwave Threshold at Storage Energy:

$$Z\_over\_n_{L\_\mu\_ring} := \frac{2 \cdot \pi \cdot E_{ring} \cdot \eta \cdot \left(min\left(\sigma_p\right)\right)^2}{q \cdot I_{peak}}$$

$$Z\_over\_n_{L\_\mu\_ring} = 0.012\ ohm$$

Transverse Microwave Threshold at Injection:

$$I_{peak\_inj} := I_{peak} \cdot \frac{\sigma_{s_0}}{\sigma_{s\_inj}}$$

$$Z_{T\_\mu} := \frac{4 \cdot \sqrt{2 \cdot \pi} \cdot E_{inj} \cdot \eta_{inj} \cdot \sigma_{p\_inj} \cdot c}{4 \cdot \pi \cdot b_{pipe} \cdot f_0 \cdot q \cdot I_{peak\_inj} \cdot \beta_{ave}}$$

$$Z_{T\_\mu} = 8.503 \ \frac{Mohm}{m}$$

Transverse Mode Coupling Threshold at Injection:

$$Z_{T\_modecpl} := \frac{4 \cdot \pi \cdot \sqrt{2} \cdot E_{inj} \cdot \nu_{s\_inj}}{q \cdot I_{peak\_inj} \cdot \beta_{ave}}$$

$$Z_{T\_modecpl} = 30.588 \ \frac{Mohm}{m}$$

THE FOLLOWING PAGES DEAL WITH CORRECTION SYSTEMS, SPARSE CORRECTORS, ALLOWABLE MULTIPOLES, ETC ETC ETC (WORK VERY MUCH IN PROGRESS!)

PLOT CELL PARAMETERS

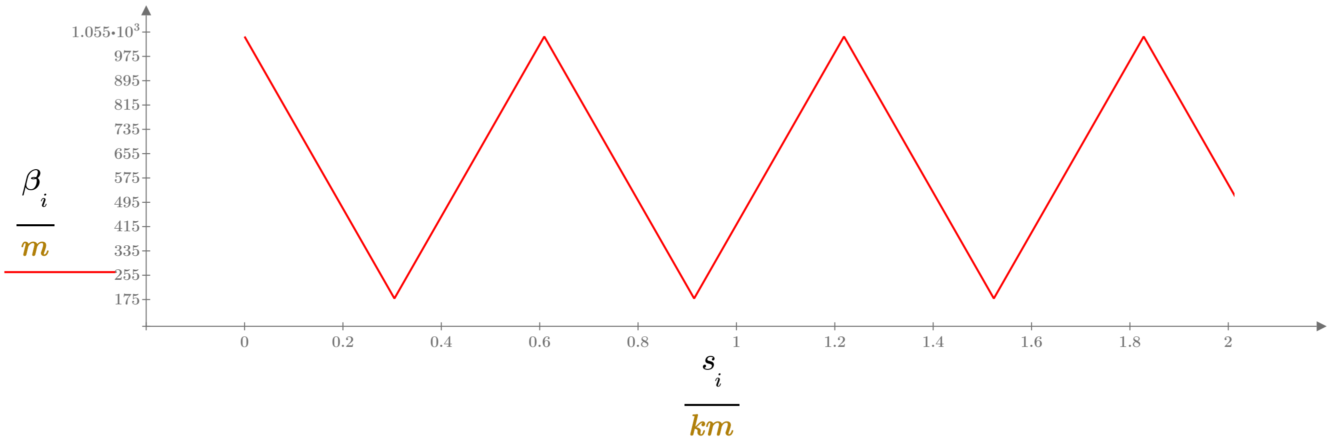
$$i := 0, 1 .. 2 \cdot \frac{L_{ArcReg}}{L_{halfcell}} \qquad j := 0, 1 .. 2 \cdot \frac{L_{ArcReg}}{L_{halfcell}}$$

$$\beta_i := \text{if}\left(\text{ceil}\left(\frac{i}{4}\right) = \frac{i}{4}, \beta_{max}, \text{if}\left(\text{ceil}\left(\frac{i}{2}\right) = \frac{i}{2}, \beta_{min}, \beta_{ave}\right)\right)$$

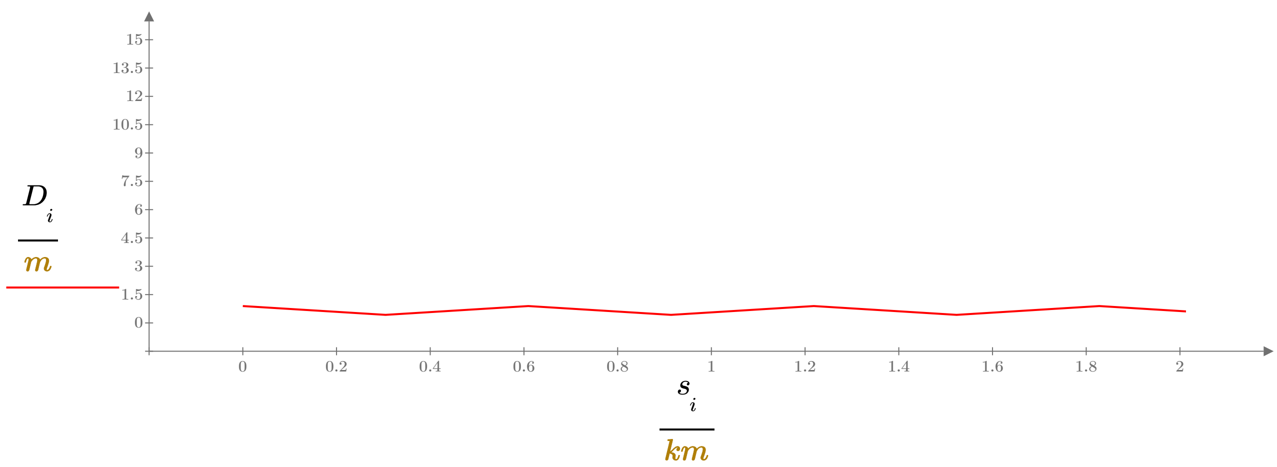
$$f_i := \text{if}\left(\text{ceil}\left(\frac{i}{4}\right) = \frac{i}{4}, F, \text{if}\left(\text{ceil}\left(\frac{i}{2}\right) = \frac{i}{2}, -F, 0 \cdot m\right)\right) \qquad s_j := \frac{j \cdot L_{halfcell}}{2}$$

$$D_i := \text{if}\left(\text{ceil}\left(\frac{i}{4}\right) = \frac{i}{4}, D_{max}, \text{if}\left(\text{ceil}\left(\frac{i}{2}\right) = \frac{i}{2}, D_{min}, D_{ave}\right)\right)$$

CELL BETA FUNCTIONS



CELL DISPERSION FUNCTIONS



ORBIT CORRECTION

Put in some orbit errors due to Quadrupole misalignments, d, and due to dipole magnet errors (roll, dB/B, etc) and compute resulting trajectory through one section between service areas.

$d_{rms} := 0.25 \cdot mm$ $\theta_{qrms} := \frac{d_{rms}}{F}$ $\theta_{qrms} = 1.16\ \mu r$ $dB_{rel} := 10^{-3}$

Assume we lump all dipoles into a SINGLE dipole error in the middle of the half cell (for even MORE simplicity):

$$\theta_{brms} := \sqrt{N_{dipolehalfcell}} \cdot dB_{rel} \cdot \theta_{dipole} \qquad \theta_{brms} = 1.083\ \mu r$$

Generate Gaussian Random Numbers:

$$A_{r_i} := \mathrm{rnd}(1) \qquad B_{r_i} := \mathrm{rnd}(1) \qquad u_i := \sqrt{-2 \cdot \ln\left(A_{r_i}\right)} \cdot \sin\left(2 \cdot \pi \cdot B_{r_i}\right)$$

$$\theta_i := \mathrm{if}\left(\mathrm{ceil}\left(\frac{i}{2}\right) = \frac{i}{2}, \frac{d_{rms}}{f_i} \cdot u_i, \theta_{brms} \cdot u_i\right)$$

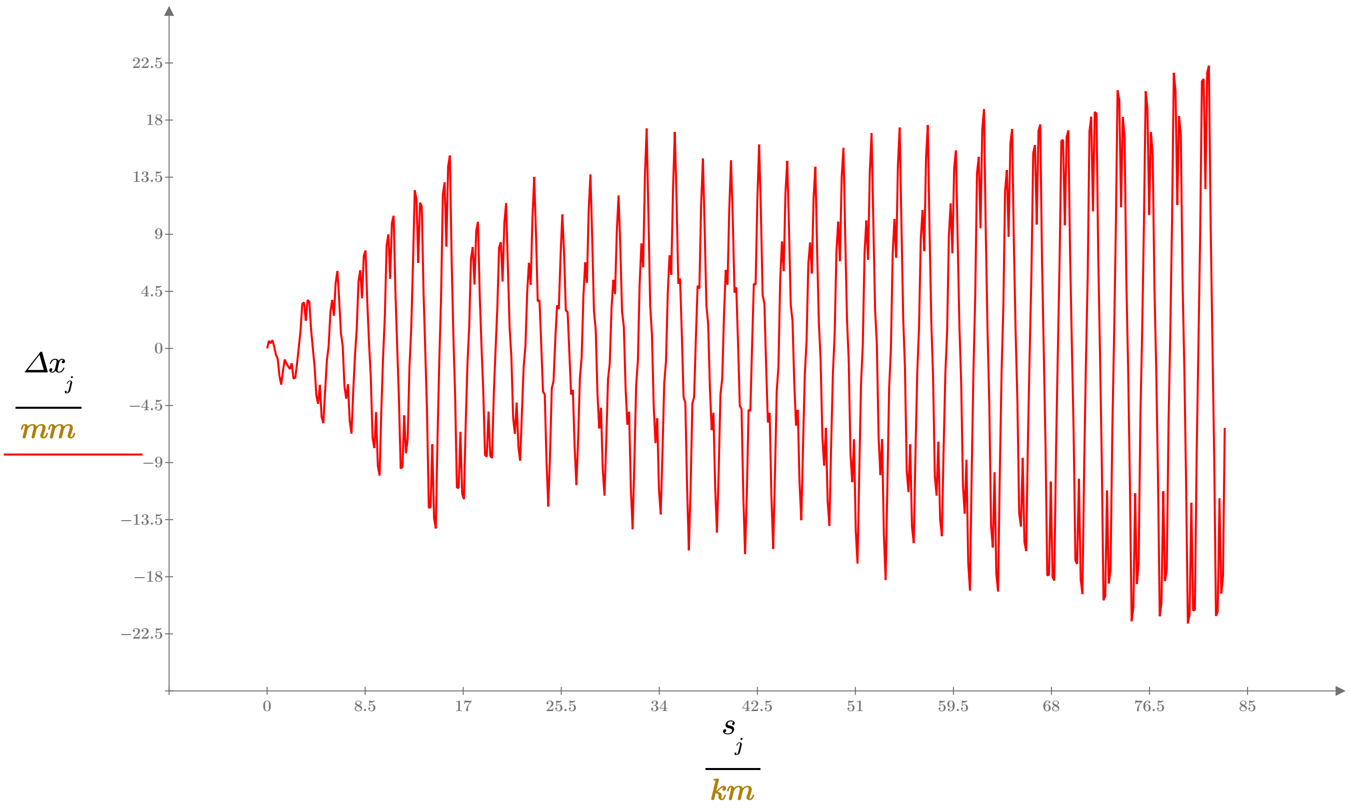
Calculate the Orbit Distortion through an arc "section" (i.e., between service areas):

$$\Delta x_j := \sum_i \left(\theta_i \cdot \sqrt{\beta_i \cdot \beta_j} \cdot \sin\left((j-i) \cdot \frac{\mu_{cell}}{4}\right) \cdot (i \le j)\right)$$

$$\max\left(\begin{bmatrix}\max(\Delta x)\\ |min(\Delta x)|\end{bmatrix}\right)=22.29\ mm$$

$$\text{mean}(\Delta x)=-0.016\ mm$$

$$\text{stdev}(\Delta x)=9.917\ mm$$



Note: might expect a typical maximum to be:

$$\Delta Xmax_{rms} := \sqrt{{\theta_{brms}}^2 + {\theta_{qrms}}^2} \cdot \sqrt{\beta_{max} \cdot \beta_{ave}} \cdot \sqrt{\frac{L_{ArcReg}}{2 \cdot L_{halfcell}}}$$

$$\Delta Xmax_{rms} = 14.759\ mm$$

Typical correction element strength to correct the above orbit distortion:

$$\Theta_{corr\_rms} := \sqrt{1+\alpha^2} \cdot \frac{\Delta Xmax_{rms}}{\beta_{max}}$$

$$\Theta_{corr\_rms} = 37.076 \ \mu r$$

Thus, maximum corrector strength: $BL_{dipcorr} := 3 \cdot \Theta_{corr\_rms} \cdot B\rho$

$$BL_{dipcorr} = 92.755 \ tesla \cdot m$$

However, if correct locally using dipole correctors at each quad, or using "moveable" quads, would find:

$$\Delta X_{rms} := \frac{1}{2 \cdot |\sin(\pi \cdot \nu)|} \cdot \sqrt{\theta_{brms}{}^2 + \theta_{qrms}{}^2} \cdot \sqrt{\beta_{max} \cdot \beta_{ave}} \cdot \sqrt{\frac{N_{halfcells}}{2}}$$

$$\Delta X_{rms} = 44.69 \ mm$$

(if left uncorrected)

but the corrector strength would be:

$$\Theta_{corr\_rms} := \sqrt{\frac{2 \cdot \beta_{ave}}{\beta_{max} \cdot \sin(\mu_{cell})^2}} \cdot \sqrt{\theta_{brms}{}^2 + \theta_{qrms}{}^2}$$

$$\Theta_{corr\_rms} = 1.718 \ \mu r$$

$$BL_{dipcorr} := 3 \cdot \Theta_{corr\_rms} \cdot B\rho$$

$$BL_{dipcorr} = 4.299 \ tesla \cdot m$$

or, if corrected by moving quadrupoles:

$$d_{corr\_rms} := \Theta_{corr\_rms} \cdot F$$

$$d_{corr\_rms} = 0.37 \ mm$$

Misalignment over time: $A := 10^{-5} \cdot \mu m^2 \cdot m^{-1} \cdot sec^{-1}$ $T := 1 \cdot yr$

$$\Delta x_{rms} := \sqrt{A \cdot T \cdot L_{halfcell}}$$

$$\Delta x_{rms} = 0.31 \ mm$$

TUNE CORRECTION

Suppose quads are built to within 1% of desired B'L, including saturation wrt dipoles... $q_{rms} := \frac{1}{F} \cdot 0.01$

$$\Delta\nu_{rms} := \frac{1}{4 \cdot \pi} \cdot \beta_{ave} \cdot q_{rms} \cdot \sqrt{N_{halfcells}} \qquad \Delta\nu_{rms} = 0.174$$

$\Delta\nu_{range} := 3 \cdot \Delta\nu_{rms} + 2$ **(+/- 2 units tuning range)** $\Delta\nu_{range} = 2.523$

Quadrupole corrector strength, assuming sparse correctors:

$$q_{corr} := \frac{4 \cdot \pi \cdot \Delta\nu_{range}}{\beta_{max} \cdot (N_{SA} \cdot 2 \cdot 4)}$$

$GL_{qcorr} := q_{corr} \cdot B\rho$ **if** $L_{qcorr} := 2 \cdot m$ **, then** $G_{qcorr} := \frac{GL_{qcorr}}{L_{qcorr}}$

$$GL_{qcorr} = 317.722 \ tesla$$

$$G_{qcorr} = 158.861 \ \frac{tesla}{m}$$

rms beta function distortion from quad errors:

$$\delta_{\beta rms} := \frac{1}{2 \cdot \sin(2 \cdot \pi \cdot \nu)} \cdot q_{rms} \cdot \beta_{max} \cdot \sqrt{\frac{N_{halfcells}}{2}}$$

$$\delta_{\beta rms} = -1.35$$

(Δβ/β)

local beta function distortion from tune correction (for tune change of one unit):

$$\delta_{\beta local} := \frac{4 \cdot \pi}{\beta_{max} \cdot (N_{SA} \cdot 2 \cdot 4)} \cdot \beta_{max}$$

$$\delta_{\beta local} = 0.157$$

(Δβ/β)

CHROMATICITY CORRECTION

average b_2 at injection:

$$b\_2_{sys} := 1 \cdot 10^{-4} \cdot \frac{1}{cm^2} \qquad \Delta\xi_{b2} := \beta_{ave} \cdot D_{ave} \cdot b\_2_{sys}$$

$$\Delta\xi_{b2} = 402.218$$

correction range: $\Delta\xi_{tot} := -\xi_{nat} + \Delta\xi_{b2} + 10$ $\qquad \Delta\xi_{tot} = 1291.845$

Sextupole strengths:

$$S_F := \frac{\pi}{\beta_{ave} \cdot D_{max}} \cdot \Delta\xi_{tot} \cdot \frac{1}{N_{SA} \cdot 2 \cdot 4} \qquad S_F = 0.093 \; \frac{1}{m^2}$$

$$S_D := \frac{\pi}{\beta_{ave} \cdot D_{min}} \cdot \Delta\xi_{tot} \cdot \frac{1}{N_{SA} \cdot 2 \cdot 4} \qquad S_D = 0.195 \; \frac{1}{m^2}$$

at 1 cm, $BL_{sxt} := S_D \cdot B\rho \cdot (1 \cdot cm)^2$ $\qquad BL_{sxt} = 16.27 \; tesla \cdot m$

TOLERABLE DIPOLE MAGNET SYSTEMATIC ERRORS (as function of half cell length)

$r_{ref} := 1 \cdot cm$

$$c_a(n) := \frac{1}{2 \cdot \pi} \cdot \int_0^{2 \cdot \pi} \cos(\psi)^n \, d\psi \qquad c2_a(n) := \frac{1}{2 \cdot \pi} \cdot \int_0^{2 \cdot \pi} \cos(\psi)^{n+1} \, d\psi$$

n = multipole order

k = number of sigma in amplitude, both transverse and longitudinal: $k := 3$

bn = multipole coefficient measured at radius rref: By = B_0 Σ (bn/rref^n) x^n

rms beam size at injection:

$$a_0(L) := \sqrt{\frac{3.41 \cdot L \cdot \varepsilon x_{n_0}}{\pi \cdot \gamma_{inj}}}$$

scaling laws:

$$\theta_d := \frac{L_{dipole}}{\rho} \qquad \beta_a(L) := \frac{L}{\sin\left(\frac{\mu_{cell}}{2}\right) \cdot \cos\left(\frac{\mu_{cell}}{2}\right)^2}$$

$$N_d := \frac{2 \cdot \pi}{\theta_d} \qquad D_a(L) := \frac{L^2}{\rho \cdot \sin\left(\frac{\mu_{cell}}{2}\right)^2}$$

Tune shift due to multipole n, and particle with m sigma amplitude:

$$\Delta\nu(n, b_n, L) := \left(\frac{\beta_a(L)}{r_{ref}}\right) \cdot \frac{N_d \cdot \theta_d \cdot b_n \cdot k^{n-1}}{2 \cdot \pi} \cdot \left(\frac{a_0(L)}{r_{ref}}\right)^{n-2} \cdot \left(\frac{n \cdot D_a(L) \cdot \sigma_{p\_inj} \cdot c_a(n) + a_0(L) \cdot c2_a(n)}{r_{ref}}\right)$$

Solve for maximum allowable bn: $\delta\nu_{max} := 0.005$ $\sigma_{p\_inj} = 4.695 \cdot 10^{-5}$

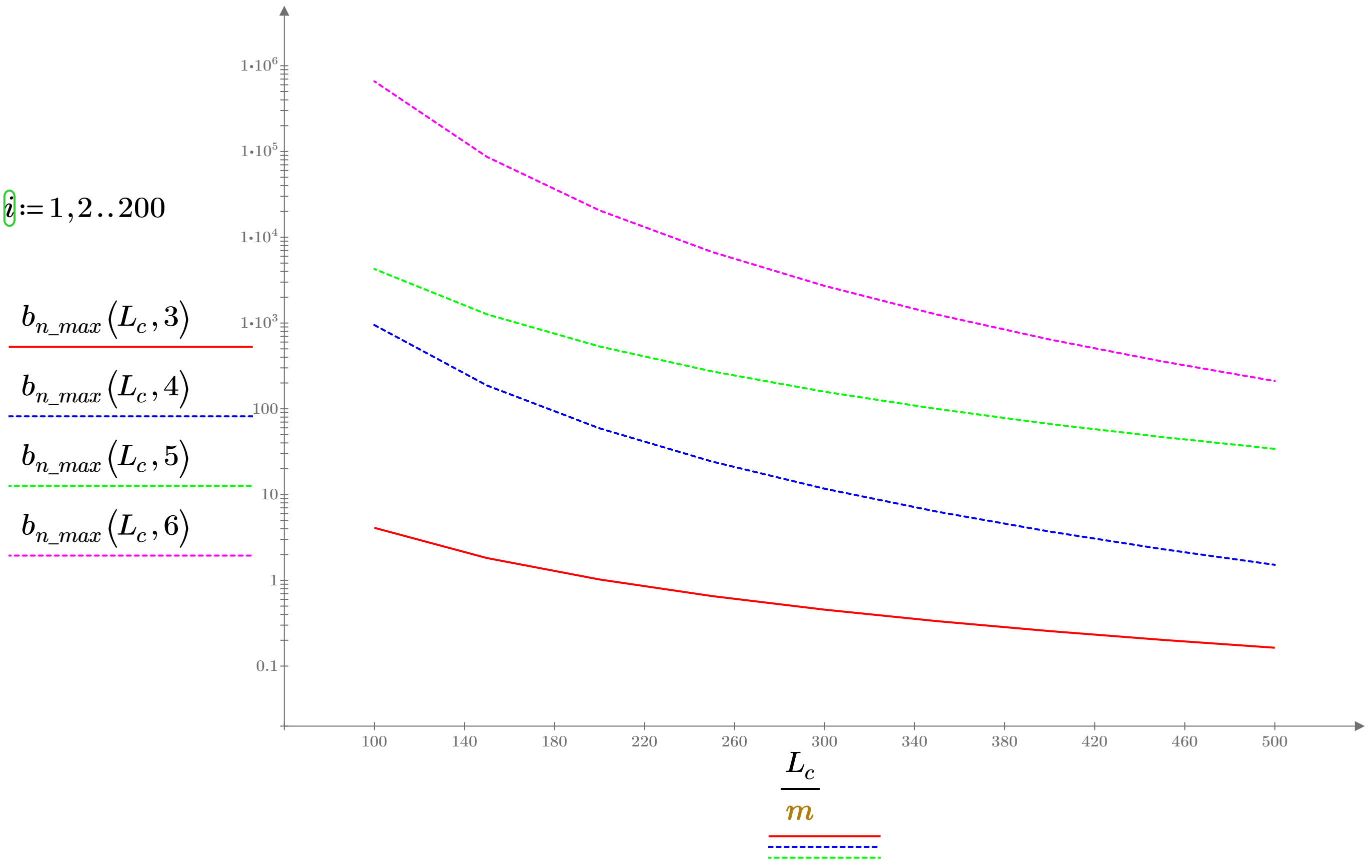
$$b_{n\_max}(L, n) := \frac{2 \cdot \pi \cdot \delta\nu_{max}}{N_d \cdot \theta_d \cdot k^{n-1}} \cdot \left(\frac{r_{ref}}{\beta_a(L)}\right) \cdot \left(\frac{r_{ref}}{a_0(L)}\right)^{n-2} \cdot \left(\frac{r_{ref}}{n \cdot D_a(L) \cdot \sigma_{p\_inj} \cdot c_a(n) + a_0(L) \cdot c2_a(n)}\right) \cdot 10^4$$

(units of 10^-4

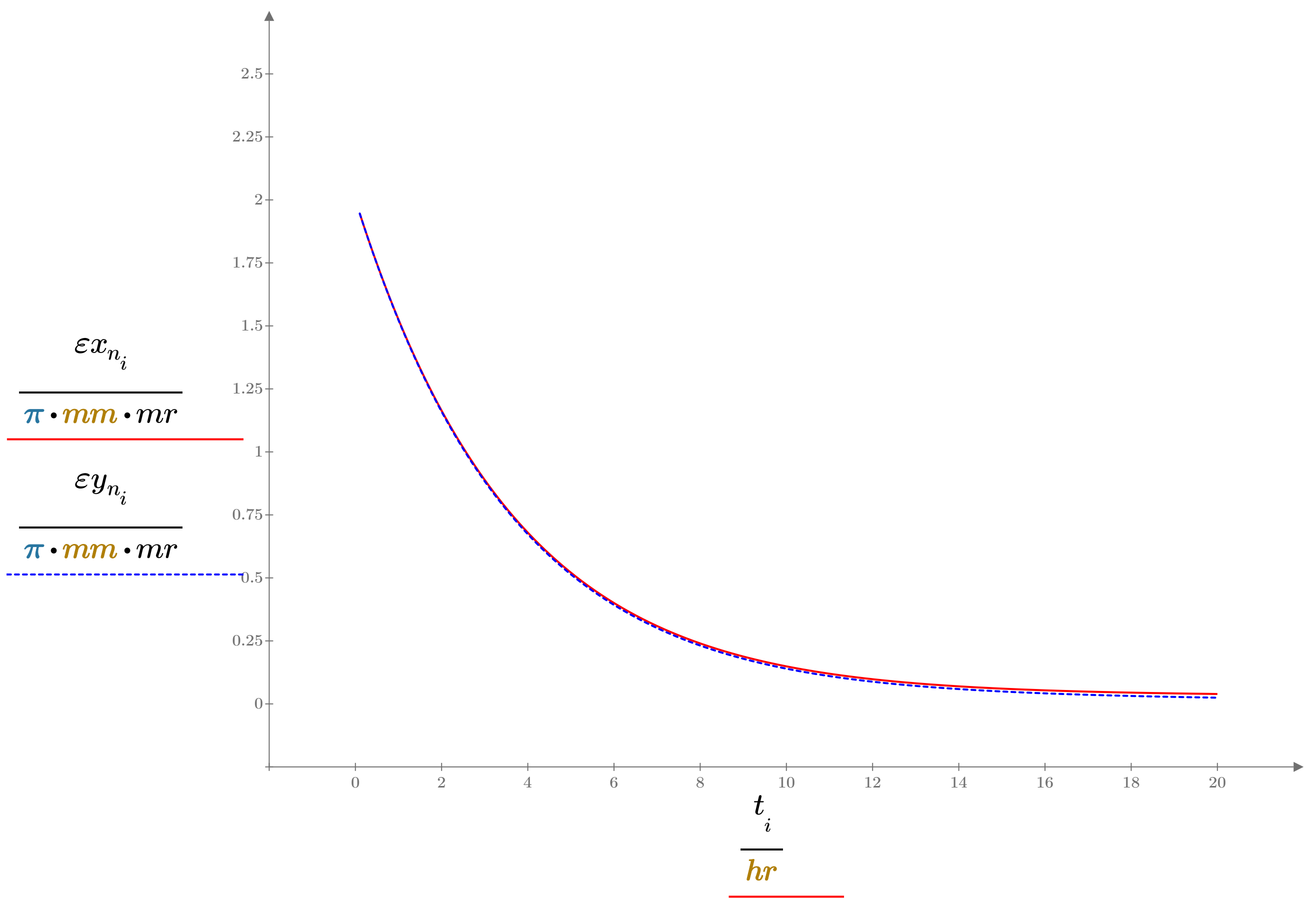
$L_c := 100 \cdot m, 150 \cdot m .. 500 \cdot m$

at $r_{ref} = 1\ cm$ **)**

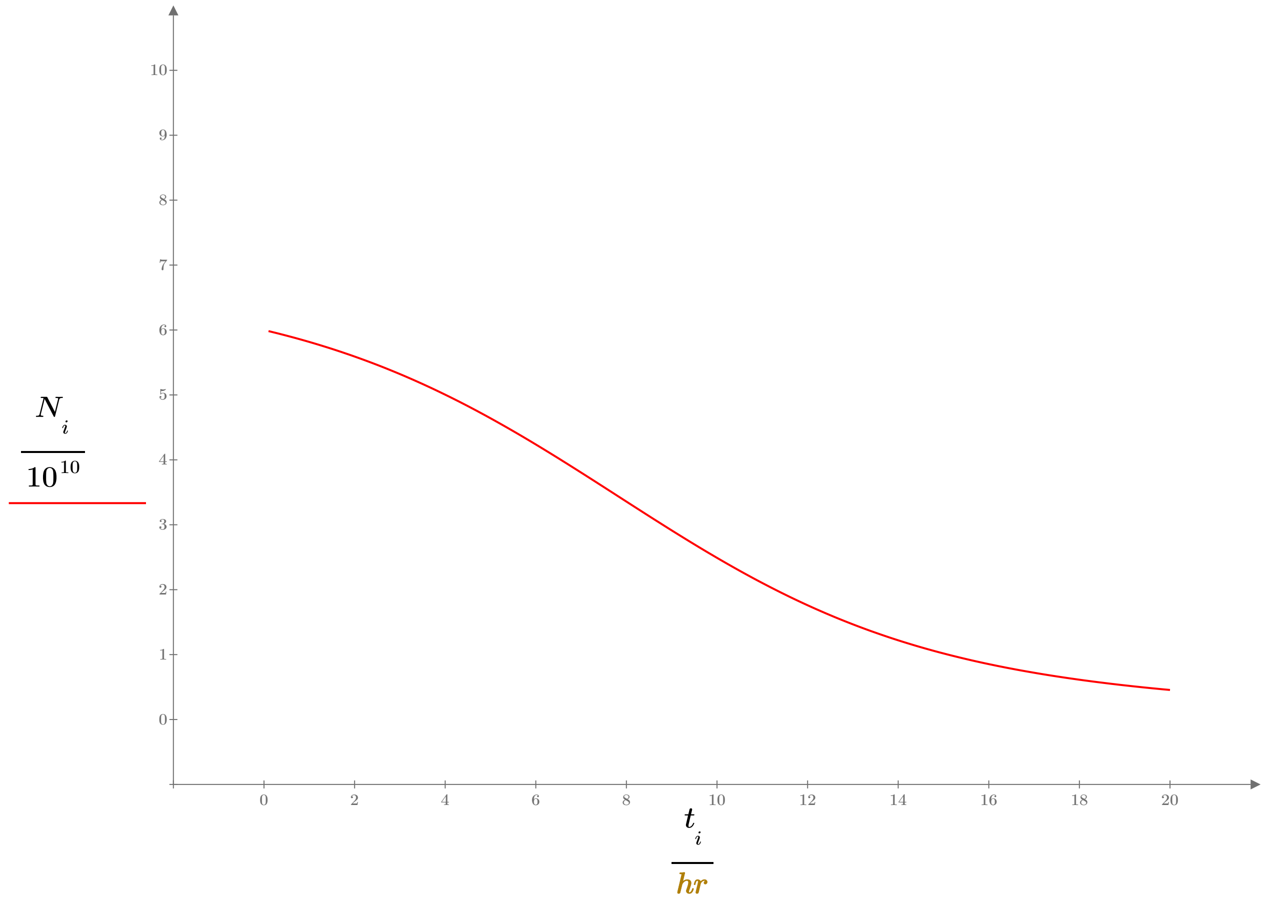
$i := 1, 2 .. 200$



Transverse Emittance vs. Time
100 TEV cm

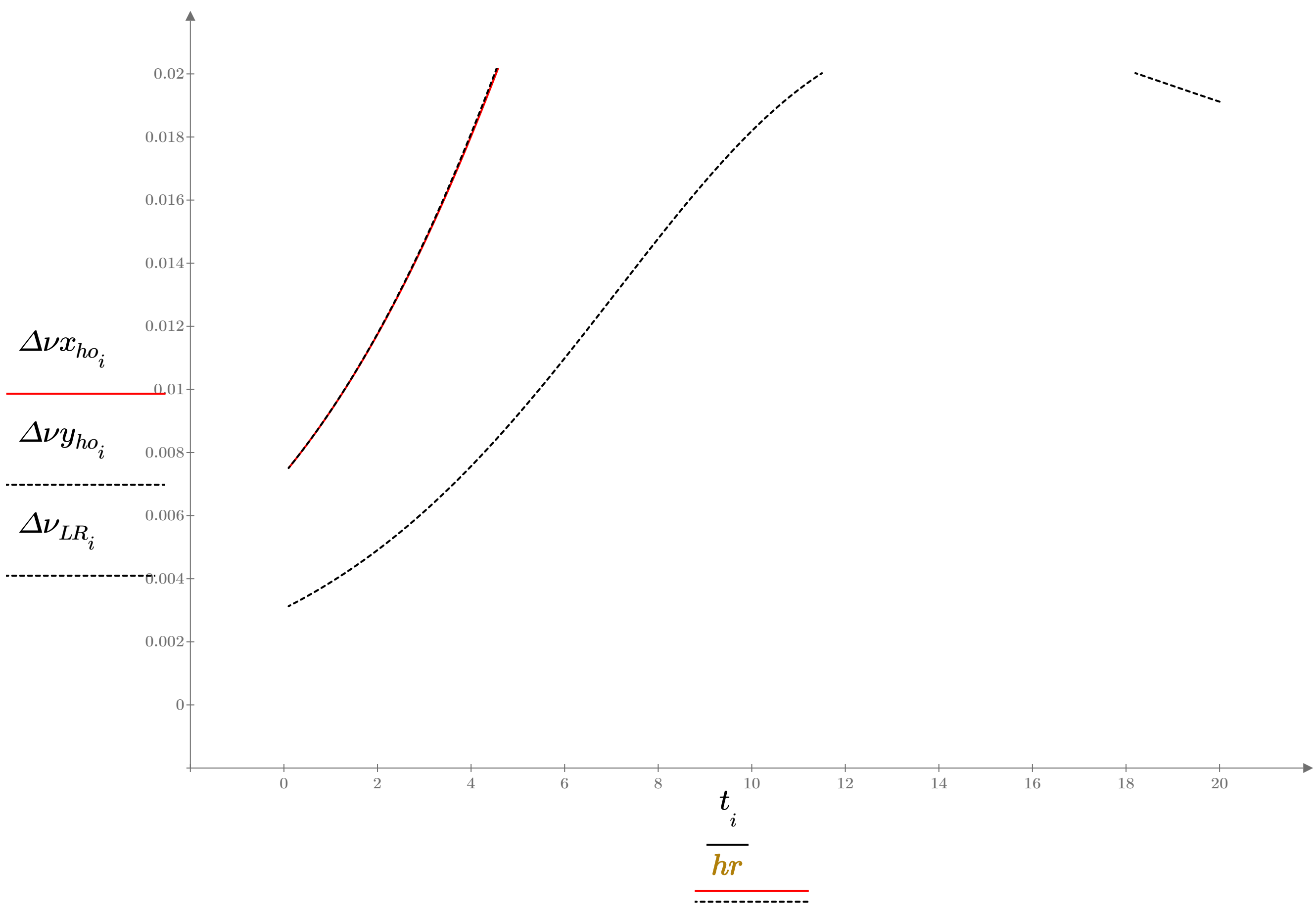


Bunch Intensity vs. Time
100 TEV cm

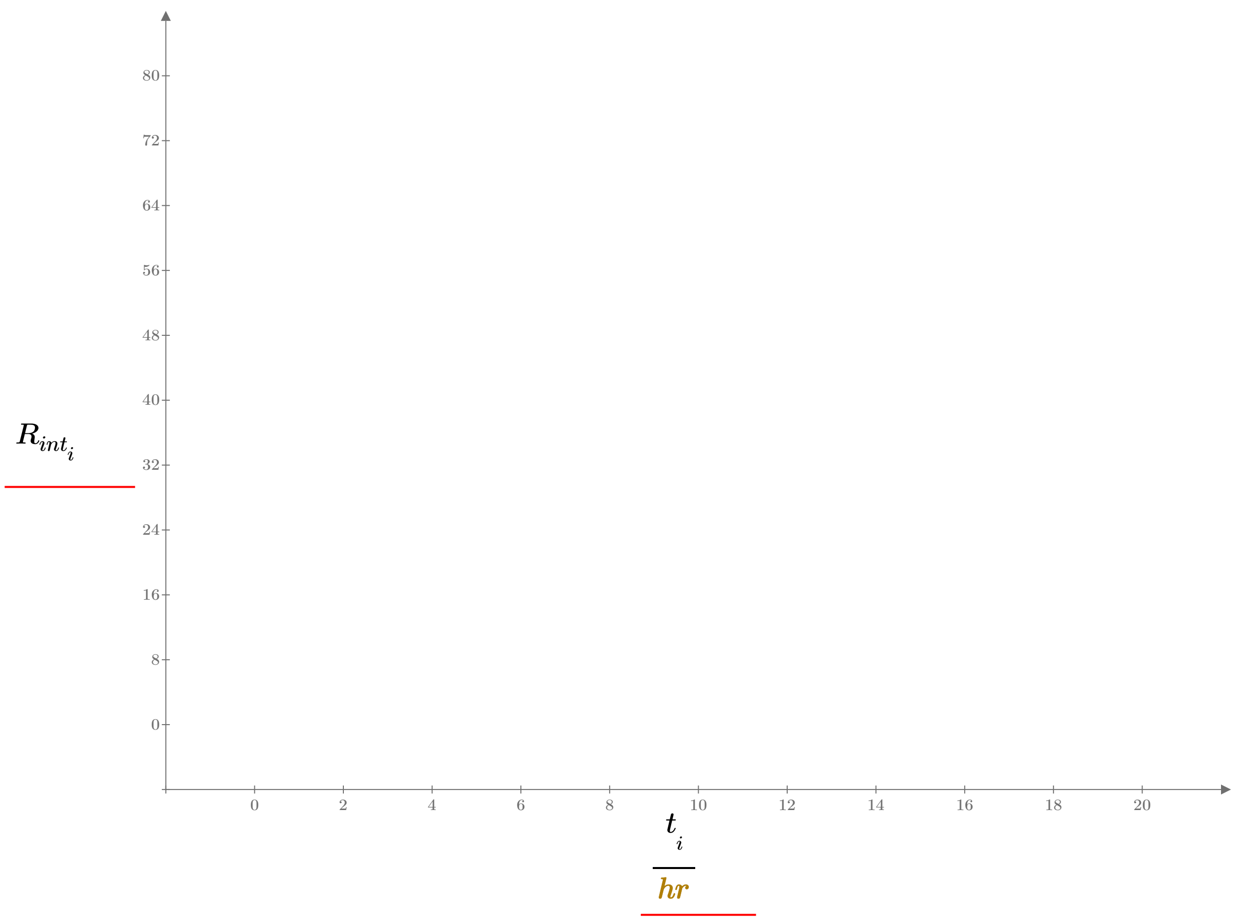


Instantaneous Luminosity vs. Time
100 TEV cm

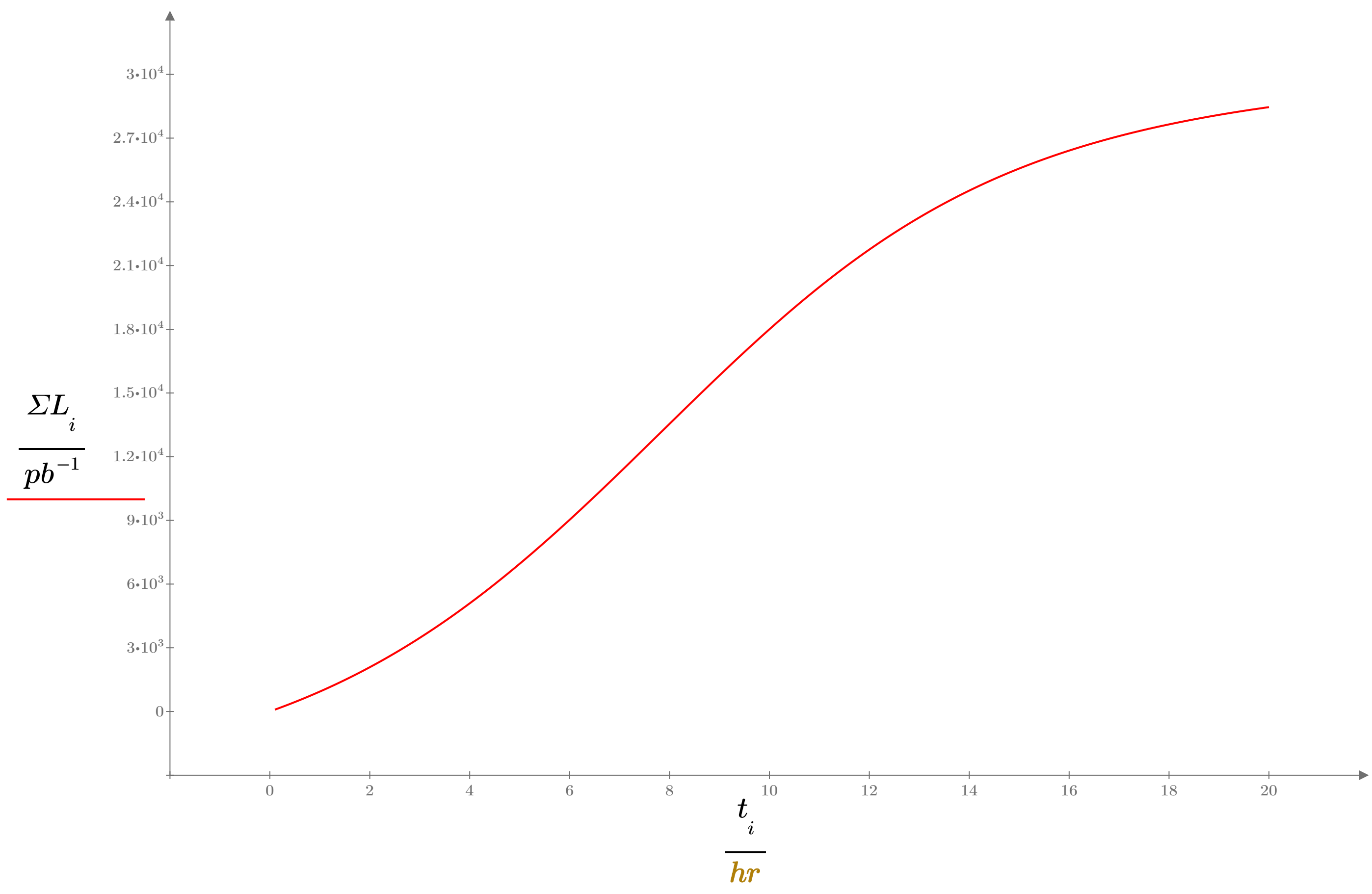
Head-on and Long Range Tune Shifts vs. Time



Interactions per Bunch Crossing vs. Time
100 TEV cm



Integrated Luminosity vs. Time
100 TEV cm



$$\varepsilon_0 := \begin{bmatrix} .5 \\ 1 \\ 1.5 \\ 2 \\ 2.5 \\ 3 \\ 3.5 \\ 4 \\ 4.5 \\ 5 \end{bmatrix} \cdot \pi \cdot mm \cdot mr$$

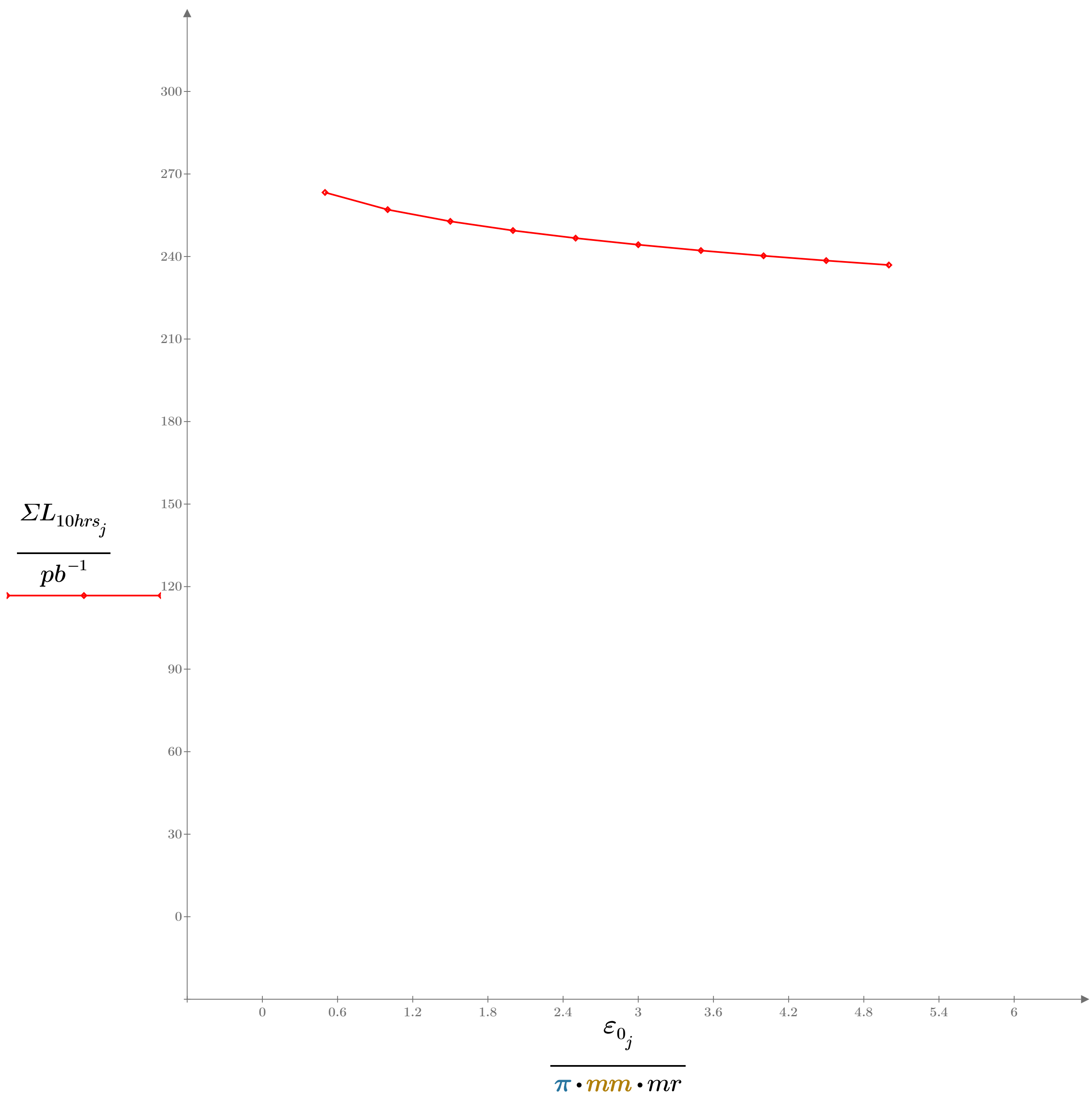
$$\Sigma L_{10hrs} := \begin{bmatrix} 263.283 \\ 257.04 \\ 252.777 \\ 249.448 \\ 246.679 \\ 244.286 \\ 242.168 \\ 240.259 \\ 238.516 \\ 236.908 \end{bmatrix} \cdot pb^{-1}$$

$$j := 0, 1 .. 9$$

$$\varepsilon x_{n_0} = 2\ \pi \cdot mm \cdot mr$$

$$\Sigma L_{100} = \left(1.8 \cdot 10^{4}\right)\ pb^{-1}$$

Integrated Luminosity (10 hr Store)
vs. Initial Transverse Emittance
100 TEV cm



$\varepsilon_{SSC} := 1$ $\varepsilon_{TEV} := 2$ $\varepsilon_{LHC} := 4$

$i := 200$

$$\begin{bmatrix} L_{i+1} \\ N_{i+1} \\ \varepsilon x_{n_{i+1}} \\ \varepsilon y_{n_{i+1}} \\ \tau p_{i+1} \\ \tau x_{i+1} \\ S_{i+1} \\ \sigma_{p_{i+1}} \\ \sigma_{s_{i+1}} \\ d_{i+1} \\ \alpha_{cross_{i+1}} \end{bmatrix} = \begin{bmatrix} \left(8.864 \cdot 10^{38}\right) \dfrac{1}{m^2 \cdot s} \\ 4.489 \cdot 10^{9} \\ \left(1.229 \cdot 10^{-7}\right) m \\ \left(7.649 \cdot 10^{-8}\right) m \\ \left(4.423 \cdot 10^{4}\right) s \\ \left(5.007 \cdot 10^{4}\right) s \\ \left(1.719 \cdot 10^{-19}\right) \dfrac{kg \cdot m^2}{s} \\ 1.914 \cdot 10^{-5} \\ 0.021\ m \\ 0.8 \\ 4.299 \cdot 10^{-6} \end{bmatrix}$$